

Estimation, Compensation and Imaging of Ultrasonic Attenuation

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Abstract Ultrasonic techniques are widely applied for diagnosis in medicine. One of the most important applications of ultrasound is to produce images of tissue structures. Diagnosis from such images is primarily based on echo size, echo pattern and the contour of organs. A major disadvantage with this technique is that certain diseases cause no change in the external contour and thus remain undetected in these images. Tissue characterization which is based directly on the ultrasonic signals has thus been an area of intense research. The attenuation of the ultrasonic signal is a feature which is of fundamental importance in tissue characterization. Two time domain estimation methods of ultrasonic attenuation are presented in Part 1. Further, a random model of ultrasonic signals is developed which provides an excellent tool for evaluation of estimation methods. Attenuation imaging is considered in Part 2. One of the time domain estimation methods is compared to a method which works entirely in the frequency domain. The methods are tested both on simulated signals with known properties and on a tissue equivalent. The time domain method is found to have less variance and to be more robust than the frequency domain estimation method. Part 3 treats the problem of compensation for the attenuation in ultrasonic signals. This compensation is traditionally performed in a Time Gain Compensation circuit, which is frequency independent. It is shown that both the resolution and the texture pattern can be improved by using time and frequency dependent compensation. A filter structure for this compensation is presented.	
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ESTIMATION, COMPENSATION AND IMAGING OF ULTRASONIC ATTENUATION

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ESTIMATION, COMPENSATION AND IMAGING
OF ULTRASONIC ATTENUATION

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GENERAL INTRODUCTION

The ultrasonic pulse echo method is a wide-spread tool in medicine and is by routine used in several fields, such as cardiology, obstetrics and gynecology. The method is based on transmitting a short radiofrequency (RF) sound burst (1-15 MHz) from a transducer into the tissue. Because of the inhomogeneities within the tissue, acoustical energy is reflected and backscattered to the transducer and registered as echoes. After compensation for the attenuation with a time gain compensation (TGC) circuit, the envelope of the reflected RF signal is calculated. By presenting the envelope on an oscilloscope, an A-mode registration is obtained which shows the echo amplitudes along the sound propagation line. Additional information can be gathered from a B-mode image where the echo amplitudes of a whole tissue area are presented on a video equipment using grey-scale technique. Such an image is created by transmitting several sound bursts from different points (linear array, compound scan) or in different directions (sector scan). By transmitting the sound bursts periodically a moving real-time image of the tissue structure is obtained. This is one of the most important applications of medical ultrasound, since it is possible to study the anatomical shape and texture continuously. The images are related to the acoustic properties of the tissue and are therefore different from images obtained from other imaging techniques, such as traditional X-ray, nuclear magnetic resonance and computer tomography. There is today some competition between these methods, where ultrasonic imaging has the advantage of combining low cost technology with non-ionizing radiation. The ultrasonic images mainly depict the anatomical structure, such as tissue boundaries, whereas information of other features, such as attenuation and velocity of the ultrasonic wave can only be acquired indirectly. In a conventional B-scan image the presence of a disease is indicated by e.g. altered appearance of anatomical structure or by abnormal motion. Sometimes the texture of the tissue can be used to conclude the presence of a disease. The discrimination is highly dependent on equipment parameters as well as the skill and experience of the operator. This motivates the search for features that enable objective evaluation of ultrasonic signals and images. Tissue characterization has thus developed out of the field of ultrasonic imaging.

Tissue characterization aims at defining those features of the ultrasonic signals and the acoustic wave propagation which characterize the condition

of the tissue. The goal is to provide quantitative information about the state of a tissue, which is not visible in a usual B-mode image. For example, it is highly probable that a diffuse disease of an organ causes no changes in the external contour of the organ and thus remains undetected in a B-mode image. Several papers have been published which treat the problem of defining features suitable for objective, quantitative discrimination. Extensive theoretical analysis and measurements of ultrasound scattering properties of tissue have been performed (SH-76, RE-76, RE-79, HI-76, HI-81, CH-77, OD-81, OD-83). Methods for different quantitative statistical analysis of the A-mode registrations have been examined (HI-81, JO-79) as well as analysis and image processing directly on the B-scan images (BA-80, FL-81, WA-83). Other studies have concentrated on the classical acoustic properties such as absorption, velocity, reflectivity and acoustic impedance (JON-80, TR-82).

The attenuation is that quantity for ultrasonic tissue characterization which has received the widest interest. The attenuation is a complex quantity which contains all the amplitude reducing effects the tissue has on the ultrasonic wave, such as absorption, scattering and beam divergence. Measurements of ultrasonic attenuation have been presented in several reports (WE-77). Although substantial variation in measurement results have been obtained, it is obvious that the attenuation of ultrasound in human tissue is frequency dependent and that higher frequencies are attenuated more than lower ones. More specifically, the attenuation has been found to increase almost linearly with both depth and frequency for a large class of tissues (PA-71, WE-77, PO-80). Several methods to estimate the attenuation have been developed; signal processing is performed on the reflected signals either in the time domain or in the frequency domain. Most efforts have been concentrated on frequency domain methods. Due to the frequency-dependent attenuation the power spectrum of the received signal shows a downward shift of its centre frequency as a function of depth. Investigations of the shift have been done assuming Gaussian power spectrum of the incident pulse from the transducer and estimation methods based on these investigations have been published (KU-79, OP-82, NA-83). The restriction of a Gaussian frequency content of the incident pulse has been loosened by other investigators (FI-83, SHA-84). The results of the methods based on the shift of centre frequency are promising and have been implemented in commercial ultrasonic equipments. A more straightforward frequency domain method is based on spectral differences: two intervals of

the reflected RF signal are gated out and an estimate of the attenuation as a function of frequency is obtained by dividing the two corresponding spectra (BE-83, KU-84). In the time domain, the zero-crossing intensity of the RF signal have been investigated (FL-83, NA-84); a property which is coupled to the centre frequency of a signal (PAP-65). Hence, the zero-crossing intensity methods estimate essentially the same properties of the tissue that are obtained by the centre frequency downshift methods in the frequency domain.

Clinical results are sparse but have been reported from liver tissue. (The liver is the largest organ of the body and is easily accessible for ultrasonic examination.) Good correlation has been found between liver cirrhosis and the attenuation (KU-80). Further, it has been documented that liver cirrhosis results in an increased echogenicity, thus implying an increase in tissue backscatter (WA-77). The eye and muscles are other organs which have been investigated in vivo (IR-82, OPM-82).

This thesis consists of three parts. In Part one, a flexible random model of an ultrasonic signal is developed by which a broad class of signals with well-known properties can easily be simulated. The model provides an excellent tool for evaluation of different estimation methods. Such an evaluation is naturally not possible when real signals measured in vivo or in vitro are used. When analyzing ultrasound signals from tissue in vivo, one is faced with the problems of an uncontrollable (usually unknown) geometry, unknown propagation paths through overlying tissue and motion of the desired tissue volume due to respiration and blood flow. In addition, biological tissue is organized randomly so that echoes returned from different sample volumes of a tissue will fluctuate randomly. A stochastic approach has thus been chosen for the generation of model signals. By means of the model, two time domain estimation methods of the attenuation are evaluated which are computationally feasible. This property is essential in order to enable attenuation imaging of tissues. Both methods operate in the time domain avoiding the computational load of multiple Fourier transformations.

In Part two, one of the time domain methods is further analyzed. It is shown that the method can be interpreted as an approximation of the linear minimum

variance estimator of the attenuation at the mean centre frequency of the received signal. The method is compared to a frequency domain method based on spectral differences. Using simulated signals both methods are able to visualize the attenuation in a model tissue. The frequency domain estimates are, however, degraded by large variance. By also taking the computational load into account, the time domain method is the most attractive. This result is further emphasized after applying the two methods on real signals received from a phantom.

Part three deals with the resolution in ultrasonic images. A common method used to increase the resolution is to make the transmitted transducer pulse shorter either by filtering techniques or by careful manufacturing of the transducer (MAR-80, MA-81, PE-80). Here, the interest has instead been focused on the gradual loss of resolution with depth caused by the frequency-dependent attenuation. In order to compensate for the attenuation the received signal is commonly amplified in a TGC. The TGC is a frequency independent device, such that all frequencies are equally amplified. Since the resolution is proportional to the bandwidth, the use of a TGC leads to degraded resolution caused by the attenuation. A compensation method based on time-varying digital processing is proposed where the compensation is both frequency and depth dependent in order to preserve the resolution. The processor consists of a sequence of filters, each of which is matched to compensate for the attenuation at a certain depth in the tissue. By switching between the filters in the sequence, time-varying processing is obtained. The effects of using time compensation and frequency dependent compensation are studied on computer simulated tissues, obtained by means of the model presented in Part one. The results show that frequency-dependent compensation improves the resolution in ultrasonic images; the improvement is considerable when the signal-to-noise ratio of the received signal is high.

The work on time domain estimation methods has previously been presented in (SA-84, CL-85). Part two and three of the thesis have been presented in part in (CL-84, CLA-85). Part two has been submitted for publication in Ultrasonics.

ESTIMATION OF THE ATTENUATION OF ULTRASONIC SIGNALS

PART 1

ABSTRACT

A flexible parametric random model of an ultrasonic signal is presented, by means of which a broad class of signals with well-known properties can be simulated. The model is used to examine two time domain methods for estimating attenuation in soft tissue. It is shown that the estimates of attenuation given by the methods are in good agreement with true attenuation values and they are robust with respect to large deterministic echoes in the ultrasonic signal. As the processing of the observed signal is performed exclusively in the time domain, low complexity calculation algorithms are achieved. Results obtained indicate that the methods work well on both broad-band and narrow-band signals.

1. INTRODUCTION

The quality, and thus the diagnostic value, of ultrasound images has increased substantially thanks to e.g. the grey-scale technique and improved resolution. These images mainly outline the structure of the tissue, whereas information about the attenuation can be acquired only indirectly from the image. This indirect interpretation of an image is very much dependent on the skill and experience of the operator.

It is desirable to discover features of an ultrasonic signal which enable discrimination between healthy and diseased tissue. A feature of particular interest is the attenuation of the ultrasonic wave in a medium. Today, there is considerable knowledge about the magnitude of the attenuation in different tissues through contributions from many investigations (PA-71, DU-76, WE-77). It is known, for example, that attenuation increases with the amount of collagen and decreases with the amount of water (PO-80). Studies of infarcts indicate an increase of collagen in the tissue, and it has been found that attenuation in infarcted tissue is larger than in the surrounding tissue. On the other hand, a cyst containing a large percentage of water has a lower attenuation than the surrounding tissue. An estimate of the attenuation in an organ can thus be of great diagnostic value.

Several experimental and theoretical contributions have been made that estimate the attenuation of a reflected signal (KU-76, KU-79, FI-83). A common method is to select segments of the reflected signal corresponding to certain parts of the organ and then estimate the attenuation between those parts by comparing the Fourier transform of the segments. Promising results from this method in clinical application have been reported (KU-80). One condition for the method to be useful is that the reflected signal is random, implying that large echoes from interfaces are not tolerated in the investigated area.

The present paper describes methods for estimating attenuation in tissue containing specular reflectors, i.e. where the reflected signal contains large echoes. As a first step a flexible parametric random model of an ultrasonic signal is developed, by means of which ultrasonic signals with well-known properties can easily be simulated. A broad class of signals can be generated by this model. It is an excellent tool for evaluating e.g. the accuracy of any attenuation estimation method, since the attenuation is known in detail.

Such an evaluation is, of course, not possible when real signals measured in vivo or in vitro are used. A phantom is also unsuitable since it causes deterministic reflected signals. Hence, the model is used throughout this paper to examine methods for estimating attenuation in soft tissue. Two time-domain estimation methods are studied. It is shown by examples that by averaging over only a small number of reflected signals the method produces estimates of the attenuation which are in good agreement with the true value even in the presence of specular echoes. In addition to these properties, the time-domain algorithms are well-suited for on-line implementation since the number of computations is modest. The methods appear to be promising for imaging attenuation in organs of the body.

2. TRANSMISSION-LINE MODEL OF ULTRASOUND SIGNALS FROM SOFT TISSUE

Three important properties can be recognized in ultrasonic signals from soft tissue:

- A. The occurrence of large echoes (specular echoes).
- B. Scattered echoes.
- C. Attenuation of the reflected ultrasonic signal.

A typical ultrasound signal $z(t)$ measured in vivo from a liver is shown in Fig. 1.1. The large reflected echoes seen at $x_1, x_2, \dots$ in Fig. 1.1 arise from larger interfaces of which dimensions are large in comparison with the ultrasound wavelength. If the surface is perpendicular to the ultrasonic beam, strong echoes will return to the transducer. Apart from the angle of incidence, the strength of a specular echo is also dependent on the difference in acoustic impedance at the reflecting boundary. Repetition of the measurement under fixed conditions yields almost the same pattern of specular echoes which thus can be considered to be deterministic.

Scattered echoes arise from structures which are small as compared to the wavelength. These echoes generally have small amplitudes and give in in vivo B-mode registrations rise to speckles that behave in a random manner.

The attenuation is an amplitude reducing process originating from absorption, scattering reflection and divergence of the sound beam. The attenuation is in general considered to be frequency dependent (WE-77, PO-80).

A model which in detail describes all properties of the tissue is extremely difficult to formulate and analyze. The three above-mentioned properties are, however, considered to be minimum requirements of a meaningful mathematical model of a reflected ultrasonic signal. Hence, the problem of finding a suitable model naturally falls into modelling of:

- A. the positions $\{x_j\}$ and the amplitude $\{m(x_j)\}$ of the specular echoes and the amplitude $w(x)$ of the scattered echoes
- B. the transmission properties of the tissue and the wave propagation from the transducer into the tissue
- C. the pulse shape of the transmitted signal.

Since no a priori information about the positions of the specular echoes is available, all positions are assumed to be equally likely within an organ. This implies that the number of specular echoes are Poisson distributed and that the distance between two echoes shows an exponential distribution. Hence, the positions $\{x_j\}$ of the specular echoes are modelled as a marked Poisson point process with varying intensity $\lambda(x)$. Their amplitudes $\{m_j\}$ are random and independent of $\{x_j\}$. Further, $\{m_j\}$ is assumed to be a white sequence uniformly distributed within $[-1,1]$ (the extreme values of a reflection coefficient). Once $\{x_j\}$ and $\{m_j\}$ have been determined in this way, these are considered to be fixed, i.e. the specular echoes are deterministic.

Scattered echoes are assumed to occur at all distances. A scattering echo is obtained by superposition of echoes from all scattering objects at equal distance from the transducer within the beamlobe. Hence, the central limit theorem indicates that the distribution of the scattering echoes is almost Gaussian and, consequently, the scattering echoes $\{w(x)\}$ are modelled as a white Gaussian zero-mean sequence with variance σ_c^2 where $\sigma_c^2 < 1$. Thus, the total reflection coefficient $c(x)$ at a distance $x/2$ from the transducer is given by

$$c(x) = \sum_j m(x_j) \delta(x-x_j) + w(x) \quad (1.1)$$

where $\delta(x)$ is the Dirac impulse. It is a nonstationary Gaussian process with mean $\sum_j m(x_j) \delta(x-x_j)$ and variance σ_c^2 . The reflection coefficient $c(x)$ represents the combined influence from all of the inhomogeneities of the tissue within the beamlobe at a distance $x/2$ from the transducer. The reflection coefficients $c(x)$ depend on two parameters, λ and σ_c . In the general case these parameters can be functions of the depth x , making it possible to simulate different organs or different regions within an organ.

The medium is assumed to exert a linear influence on the propagating ultrasonic pulse. Hence, the medium is described by a linear frequency function, which also must be a function of the depth $x/2$. The two-way transfer function of the medium is denoted by $H(f;x)\exp(-j2\pi fx/v)$ where the exponent represents the delay caused by the two-way propagating time $\tau_x = x/v$ of the pulse and v [cm/ μ s] is the velocity of the sound propagation in the medium.

The velocity of the sound pulse propagating through the medium can in the general case vary with depth. However, throughout this thesis it is considered to be constant giving the simple relationship above between time and depth. $H(f;x)$ describes all the frequency-dependent properties of the tissue (the frequency dependence of the attenuation, scattering, reflection coefficient etc). For our purposes, the most essential properties of the tissue are only taken into account in order to make the model mathematically manageable. In this paper only the attenuation of the tissue is considered and it is described by the transfer function

$$H(f;x) = \exp\{-A(x)\} \exp\{-\Gamma(x)f^{\beta(x)}\} \quad (1.2)$$

where $A(x)$, $\Gamma(x)$ and $\beta(x)$ are functions which characterize the tissue. Some authors exclude the first part $\exp\{-A(x)\}$ of Eqn. (1.2) since they are only interested in the frequency dependent part of the attenuation (KU-79, KA-80, GU-82). Measurements show, however, that there is a frequency independent part of the attenuation (WE-77, PO-80). This fact is due to the loss of energy caused by the large number of reflections (HOL-78). The second factor in Eqn. (1.2), $\exp\{-\Gamma(x)f^{\beta(x)}\}$, is related to the absorption of energy in the tissue and is frequency dependent. Note that the two functions $A(x)$ and $\Gamma(x)$ describe the attenuation between the transducer and a point at the depth $x/2$. The attenuation parameters at the point $x/2$ are defined by

$$\begin{cases} a(x) = \frac{dA(x)}{dx} \\ \gamma(x) = \frac{d\Gamma(x)}{dx} \end{cases} \quad (1.3)$$

The functions $a(x)$, $\gamma(x)$ and $\beta(x)$ are considered to be constant in organs with homogeneous tissue.

A suitable two-way transducer pulse $s_0(t)$ has to be chosen. A simple approach is to use a recorded pulse from a real transducer. However, in order to enable variation of the frequency properties of the transducer, a rational approximation of a transducer pulse is used.

It remains to put together the three steps described above into a block diagram and to design an algorithm for calculating a simulated reflected signal. Note that the transducer pulse $s_0(t)$ after reflection at depth $x/2$ returns as an echo with the form $c(x)s_x(t-\tau_x)$. The Fourier transform of $s_x(t)$ is

$$S_x(f) = S_0(f)H(f;x) \quad (1.4)$$

where $S_0(f)$ is the Fourier transform of $s_0(t)$. The model of the reflected ultrasonic signal can be described in a block diagram, see Fig. 1.2. Since the calculations are performed in discrete time the reflected signal is sampled with the sampling rate $f_s = 1/T$. The echoes are considered to originate from discrete depths $x_i/2 = ifv/2$. The sampled output $z(k)$, which is a model of the sampled real reflected signal $z_0(t)$, is easily obtained by

$$z(k) = \sum_i c(i)s_i(k-i) \quad (1.5)$$

where $c(i) = c(x_i)$. The Fourier transform of $s_i(k)$ is (c.f. Eqns. (1.2) and (1.4))

$$S_i(f) = S_0(f) \exp \left\{ -A(i) - \Gamma(i) f^{\beta(i)} \right\} \quad (1.6a)$$

which for a homogeneous tissue will be simplified as

$$\begin{aligned} S_i(f) &= S_0(f) \exp \left\{ -(a + \gamma f^\beta) T v i \right\} = \\ &= S_0(f) \exp \left\{ -b(f) T v i \right\} \end{aligned} \quad (1.6b)$$

where $b(f) = a + \gamma f^\beta$ is a measure of the attenuation. The model will now be used for calculating signals reflected in a homogeneous tissue, such as liver. The medium contains both specular echoes and scatters, i.e. $c(i) = \sum_j m(j) \delta(i-j) + w(i)$, with the scattering intensity $\sigma_c(i)$ considered to be constant. Fig. 1.3 shows $c(i)$, $m(i)$ and $w(i)$. The attenuation functions $a(i)$, $\gamma(i)$ and $\beta(i)$ are considered to be constant and equal to $a(i) = 0.1 \text{ Npcm}^{-1}$, $\gamma(i) = 0.2 \text{ Npcm}^{-1} \text{ MHz}^{-1}$ and $\beta = 1$ respectively. Two different transducers, one broadband and one narrowband, have been used. Their two-way impulse responses and frequency functions are shown in Fig. 1.4. The reflected signals (Fig. 1.5) are calculated using a sampling rate of $f_s = 1/T = 20 \text{ MHz}$ and a wave propagation velocity $v = 1540 \text{ ms}^{-1}$. After a transient interval of approximately 100 samples, the simulated signal in Fig. 1.5a resemble the real signal in Fig. 1.1. Simulated reflected signals like those in Fig. 1.5 will be used in the next section to examine methods for estimating attenuation.

3. INVESTIGATION OF ULTRASONIC SIGNALS

In the preceding section it was shown that the tissue can be characterised by the model parameters $\lambda(i)$, $m(i)$, $\sigma(i)$, $a(i)$, $\gamma(i)$ and $\beta(i)$. We will now investigate methods for estimating the attenuation parameters $a(i)$, $\gamma(i)$ and $\beta(i)$ of a homogeneous tissue (where the attenuation is assumed to be constant within the examined interval). For this purpose the reflected signal (Eqn. (1.5)) must be written in an appropriate form. The Fourier transform (Eqn. (1.6b)) is therefore rewritten as

$$\begin{aligned} S_i(f) &= \exp\{-b(f_0)Tvi\} S_0(f) \exp\{-[b(f)-b(f_0)]Tvi\} \\ &= \exp\{-b(f_0)Tvi\} S'_i(f) \end{aligned} \quad (1.6c)$$

where f_0 is within the passband (f_1, f_2) of the transducer and f_1 and f_2 denote the lower and upper cutoff frequencies, respectively, of the transducer. By means of Eqn. (1.6c) the reflected signal can be written as

$$z(k) = \exp\{-b(f_0)Tvk\} u(k) \quad (1.7)$$

where

$$u(k) = \sum_i c(i) \exp\{b(f_0)Tv(k-i)\} s'_i(k-i) \quad (1.8)$$

is recognised as the convolution sum between the random sequence $\{c(i)\}$ and the time-variable impulse response $\exp\{b(f_0)Tvk\} s'_i(k)$. The first factor in Eqn. (1.7) depends solely on the attenuation in the tissue and is independent of the structure of the tissue, i.e. of the reflection coefficients $\{c(i)\}$. The signal $u(k)$ also depends on the choice of f_0 in addition to the tissue parameters.

The power of $z(k)$ is defined as its variance, i.e.

$$\begin{aligned} P_z &= V\{z(k)\} = \exp\{-2b(f_0)Tvk\} V\{u(k)\} = \\ &= \exp\{-2b(f_0)Tvk\} P_u \end{aligned} \quad (1.9)$$

By choosing f_0 such that the power P_u of $u(k)$ is independent of k , the decrease of the signal $z(k)$ is determined by the first factor in Eqn. (1.7). Then an estimate of the attenuation can apparently be obtained by measuring the decrease of $z(k)$. It is obvious from Eqn. (1.2) that higher frequencies are attenuated more than lower ones, a difference which increases with depth in the tissue. Hence the center frequency gradually decreases. A thorough investigation of the decay of the center frequency has been performed (SA-85), where it is shown that f_0 can be interpreted as the mean center frequency over the studied interval $[0, x]$. By assuming (I) $\beta=1$, (II) time-limited signals $s_i^1(k)$, (III) a slow variation of $s_i^1(k)$ with i and (IV) an idealised transducer with zero attenuation inside and infinite attenuation outside the passband, it can be shown that P_u is independent of k if

$$f_0 = f_1 + B \frac{1}{2\gamma B T \gamma k} \ln \left\{ \frac{2\gamma B T \gamma k}{1 - \exp[-2\gamma B T \gamma k]} \right\} \quad (1.10)$$

where $B = f_2 - f_1$ is the bandwidth of the transducer (SA-85). It is easily seen from Eqn. (1.10) that f_0 decreases in a nonlinear fashion for increasing k , but within a limited interval f_0 can be approximated by a constant. Further, in many situations of interest, the exact value of f_0 is not needed, such as when measurements of the relative variations of the attenuation inside an organ are studied. The parameters a , γ and β are estimated as follows: Since the value $2\gamma B T \gamma k$ in Eqn. (1.10) is small for a narrowband transducer, f_0 is approximately equal to the centre frequency f_c of the transducer, i.e.

$$f_0 \approx f_1 + \frac{B}{2} = f_c \quad (1.11)$$

An estimate of $b(f_c) = a + \gamma f_c^{\beta}$ is obtained by measuring the decrease of the reflected signal from a narrowband transducer. To estimate the parameters a , γ and β , measurements of $b(f)$ for at least three frequencies are needed. This can be performed either by using at least three narrowband transducers with different centre frequencies, or even better from a practical point of view, by using a broadband transducer and processing the reflected signal in narrowband filters with different centre frequencies. The latter method can be implemented by the short-time Fourier transform. By measuring $b(f) = a + \gamma f^{\beta}$ at N frequencies f_n , with $n = 1, 2, \dots, N$, an estimate of the individual parameters can be calculated by minimizing the error

$$c = \sum_{n=1}^N [a + \gamma f_n^\beta - b(f_n)]^2 \quad (1.12)$$

Experimental findings indicate that β should be constrained to $1 \leq \beta \leq 2$ (WE-77).

4. ESTIMATION OF ATTENUATION

We will now examine two different methods for estimating the attenuation $b(f_0)$ of the reflected signal (Eqn. (1.7)):

- (1) short-time energy (STE)
- (2) nonlinear filtering (NF).

These methods are based on the fact that the decay of $z(k)$ is $\exp\{-b(f_0)Tvk\}$ and the fact that $z(k)$ has low-frequency properties compared to $u(k)$.

Short-time Energy Method

The short-time energy of the reflected signal $z(k)$ is defined by

$$E_z(k) = \sum_n z^2(n)p(k-n) \quad (1.13)$$

where the window function $p(k)$ is zero outside the interval $[-L, L]$. The short-time energy is a measure of the energy of $z(k)$ in the time interval where $p(k-n) \neq 0$. Introducing Eqn. (1.7) into Eqn. (1.13) yields

$$\begin{aligned} E_z(k) &= \sum_n \exp\{-2b(f_0)Tvn\} u^2(n)p(k-n) \approx \\ &\approx \exp\{-2b(f_0)Tvk\} \sum_n u^2(n)p(k-n) = \\ &= \exp\{-2b(f_0)Tvk\} E_u(k) \end{aligned} \quad (1.14)$$

The approximation in Eqn. (1.14) is valid since $\exp\{-2b(f_0)Tvk\}$ varies slowly compared with $u(k)$ and since the width of the window is small. By taking the logarithm of both sides of Eqn. (1.14) we obtain

$$\frac{1}{2} \ln\{1/E_z(k)\} \approx b(f_0)Tvk + \frac{1}{2} \ln\{1/E_u(k)\} \quad (1.15)$$

The attenuation $b(f_0)$ is estimated simply by fitting a straight mean-squares line to $\frac{1}{2} \ln\{1/E_z(k)\}$. The method is illustrated in Fig. 1.6 for both a broadband and a narrowband transducer. The short-time energy decreases rapidly in both cases whereas $\ln\{1/E_z(k)\}$ apparently increases linearly.

Nonlinear Filtering Method

The second method for splitting the two factors of the observed signal is to determine the envelope of $z(k)$ by nonlinear filtering. This method has the advantage of being less complex computationally as compared to an ordinary quadrature envelope demodulator. Since the observed signal $z(k)$ is a bandpass signal we can rewrite it as

$$z(k) = \exp\{-b(f_0)Tvk\}e(k)\cos\left(2\pi f_0k+\Phi(k)\right) \quad (1.16)$$

where $e(k)$ is the envelope and $\Phi(k)$ is the phase function of $u(k)$. The signal is now processed in the circuit shown in Fig. 1.7a. The input signal to the nonlinear filter is

$$x(k) = b(f_0)Tvk - \ln e(k) - \ln |\cos(2\pi f_0k+\Phi(k))| \quad (1.17)$$

When $\cos(2\pi f_0k+\Phi(k))$ approaches zero short spikes with very large amplitudes appear. These spikes have to be filtered out before further processing is done. Since the number of samples of $z(k)$ is limited the filter function must be short in the time domain. This is achieved by using a nonlinear filter consisting of a median filter followed by a short linear smoothing filter, as shown in Fig. 1.7b. The input/output relation of a median filter with length $(2K+1)$ is given by

$$q(k) = \text{median}_{l=-K}^K \{x(k+l)\} \quad (1.18)$$

Large values of short duration will be eliminated by this filter, and the following linear filter of length $(2L+1)$ smooths the signal. In our investigations we have used very short filters, some typical values were $K=2$ and $L=2$. The output $y(k)$ is an approximation of $b(f_0)Tvk - \ln(e(k))$, i.e.

$$\begin{aligned} y(k) &= b(f_0)Tvk - \ln e(k) + \epsilon_1(k) = \\ &= b(f_0)Tvk + v(k) + m \end{aligned} \quad (1.19)$$

Thus $y(k)$ is the sum of the unknown linear function $b(f_0)Tvk$, the slope to be estimated, and the noise $v(k)+m$, where $v(k)$ is a realisation of a zero-mean random process. The attenuation factor $b(f_0)$ is now estimated

by fitting a straight line to $y(k)$ in the mean square sense. The signals $x(k)$ and $y(k)$ are shown in Fig. 1.8 along with the fitted line $\hat{b}(f_0)Tvk + \hat{m}$. By subtracting the estimated line we obtain

$$-\ln(\hat{e}(k)) = -\ln(e(k)) + \epsilon_2(k) \quad (1.20)$$

where $\epsilon_2(k)$ is the estimation error. When $\epsilon_2(k)$ is small we obtain the following approximation of the estimated envelope

$$\hat{e}(k) \approx e(k) - \epsilon_2(k)e(k) = e(k)\epsilon_3(k) \quad (1.21)$$

where $\epsilon_3(k)$ is the error in the estimated envelope. Fig. 1.9a shows the estimate $\hat{e}(k)$ for the broadband signal in Fig. 1.5. The scattering echoes can be removed by averaging over several reflected signals and thus an estimate of the envelope of the specular echoes will be obtained as seen in Fig. 1.9c. From Fig. 1.9 we note the good correspondence between this envelope and the true positions and magnitudes of the specular echoes.

5. RESULTS

A number of both narrowband and broadband ultrasonic signals have been simulated with the attenuation parameters $\alpha=0.1 \text{ Np cm}^{-1}$, $\gamma=0.2 \text{ Np cm}^{-1} \text{ MHz}^{-1}$ and $\beta = 1$ or 2 . The centre frequencies of the narrowband transducers were set to 0.95 , 1.95 and 2.90 MHz , respectively, and the band limits of the broadband transducer were $(f_1, f_2) = (1.78, 3.53) \text{ MHz}$ (c.f. Fig. 1.5). Estimates of attenuation using the two methods from Section 4 on narrowband signals ($f_c=1.95 \text{ MHz}$) are presented in Table 1.1, where the "true" values are also noted. The length of the estimation interval is 600 samples of the reflected signal, which corresponds to a length of 2.25 cm in the tissue. It is seen from Table 1.1 that the variation in the estimates is quite large and that the two methods yield almost the same results.

Signal	b_{true}	$\hat{b}_{\text{NF}}$	$\hat{b}_{\text{STE}}$
1	0.49	0.67	0.65
2	0.49	0.52	0.50
3	0.49	0.49	0.49
4	0.49	0.49	0.50
5	0.49	0.52	0.52

Table 1.1. Single estimates of attenuation for five different narrowband reflected signals with α , γ , $\beta = 0.1$, 0.2 , 1 , respectively, and $f_0 = 1.95 \text{ MHz}$.

To obtain a measure of the quality of the estimate obtained by the NF method, a number of statistical parameters of the estimate are calculated for both narrowband and broadband signals. The centre frequency of the narrowband transducer is 1.95 MHz . The properties of the broadband transducer are shown in Fig. 1.5a. The decay of the reflected signal is determined by f_0 , which for the broadband transducer is calculated from Eqn. (1.10). A good approximation of f_0 in the actual estimating interval is $f_0=2.35 \text{ MHz}$, resulting in a true attenuation equal to $b_{\text{true}} = \alpha + \gamma f_0 = 0.57 \text{ Np cm}^{-1}$. The calculated statistical parameters are summarized in Table 1.2.

The variation interval $(\hat{b}_{\text{min}}, \hat{b}_{\text{max}})$ is large for a small estimating interval. This variation influences the average value which is sensitive to a few very large or very small values of $\hat{b}_{\text{NF}}$. The individual estimates $\hat{b}_{\text{NF}}$ obtained from

narrowband signals are shown in Fig. 1.10a. The mean value $\langle \hat{b}_{NF} \rangle$ as a function of the number of averages reflected signals is shown in Fig. 1.10b, where the fast convergence is notable. Figs. 1.10c and 1.10d show the corresponding functions obtained from broadband signals. Note the smaller variation interval for the broadband transducer. A good estimate is obtained for both transducers by averaging as few as 5-10 reflected signals.

Transducer	b_{true}	Estimate interval	Average value	Standard deviation	Median	$(\hat{b}_{min}, \hat{b}_{max})$
Narrow-band	0.49	200-800	0.54	0.10	0.51	0.34, 0.76
	0.49	400-1000	0.48	0.10	0.48	0.21, 0.68
	0.49	200-1000	0.51	0.07	0.52	0.37, 0.62
Broad-band	0.57	200-800	0.60	0.07	0.59	0.40, 0.75
	0.57	200-1000	0.56	0.04	0.56	0.43, 0.66

Table 1.2. Some statistical figures of the estimates $\hat{b}(f_0)$, nonlinear filtering (50 estimates used).

To obtain an estimate of the individual parameters, a , γ and β , the attenuation at three different frequencies for five reflected signals is measured. The estimates of $\hat{b}_{NF}$ in Table 1.3 are used for calculating a , γ and β from Eqn. (1.12), with β constrained to $1 \leq \beta \leq 2$. The following values are found:

from Table 1.3a: $\hat{a} = 0.15$, $\hat{\gamma} = 0.18$, $\hat{\beta} = 1$

from Table 1.3b; $\hat{a} = 0.11$, $\hat{\gamma} = 0.19$, $\hat{\beta} = 1.95$

The agreement with the true values is good, although only a few measurements of $b(f)$ are available.

f_0	(a) $\beta = 1$			(b) $\beta = 2$		
	b_{true}	$\hat{b}_{NF}$	$\hat{b}_{STE}$	b_{true}	$\hat{b}_{NF}$	$\hat{b}_{STE}$
0.95	0.29	0.31	0.31	0.28	0.29	0.29
1.95	0.49	0.54	0.53	0.87	0.82	0.82
2.90	0.68	0.65	0.66	1.80	1.63	1.64

Table 1.3. Averaged estimates of the attenuation. (Each estimate from five reflected ultrasonic signals of various frequencies.) In both tables $(a, \gamma) = (0.1, 0.2)$.

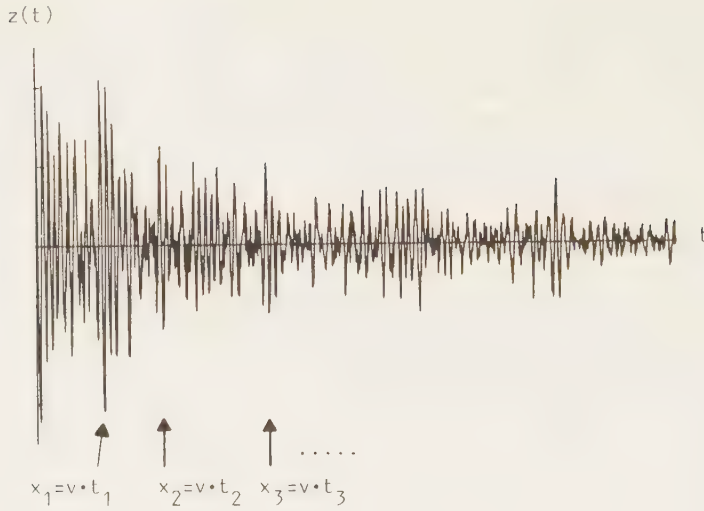


Fig. 1.1 A typical in vivo ultrasonic reflected signal from liver tissue using a 13 mm, 2.5 MHz transducer.

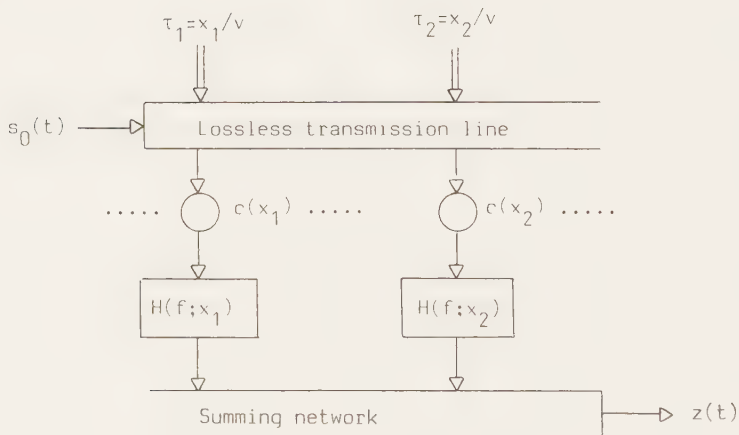


Fig. 1.2 Transmission line model of soft tissue.

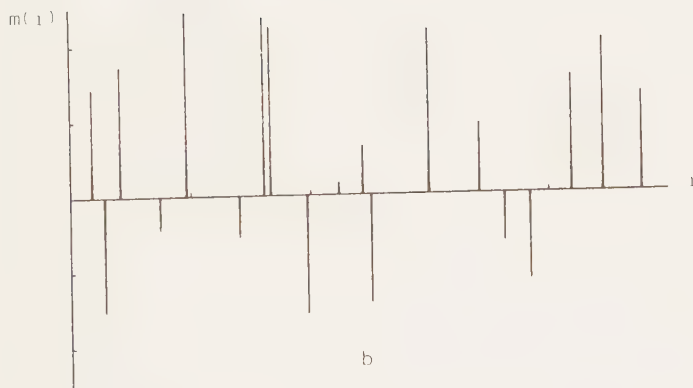
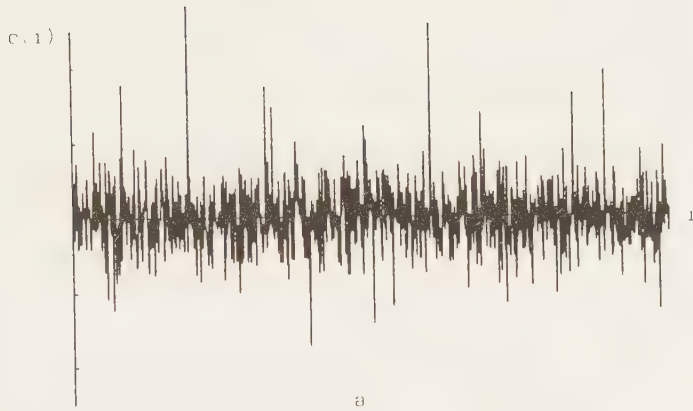


Fig. 1.5 Cont.

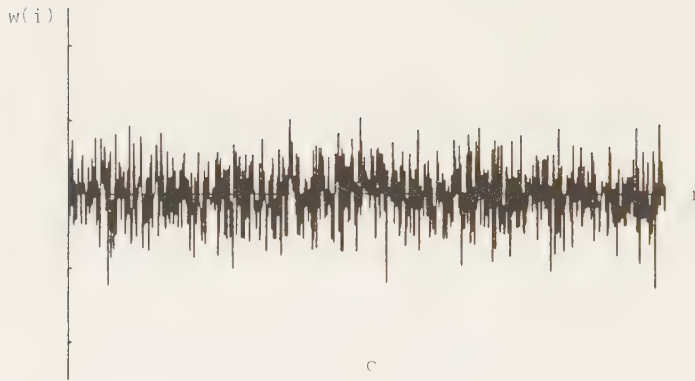


Fig. 1.3 Model of the physiological structure of the tissue:

- a. The total echo amplitude $c(i)$ composed of the two parts in Fig. 1.3b and 1.3c.
- b. The specular echo amplitude $m(i)$.
- c. The scattered echo amplitude $w(i)$.

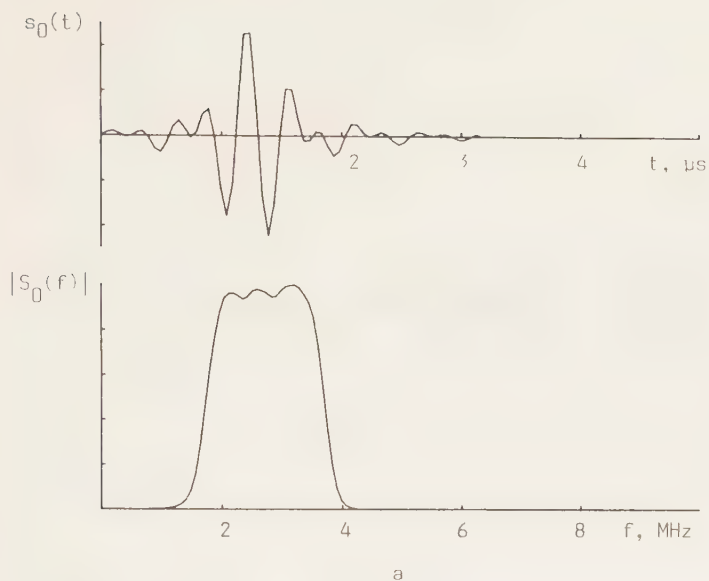


Fig. 1.4a Impulse response $s_0(t)$ and corresponding amplitude function $|S_0(f)|$ of a broadband transducer.

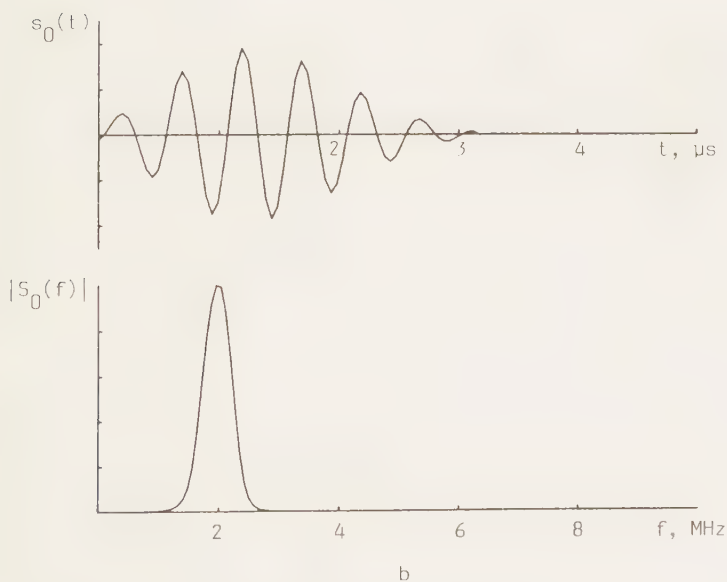


Fig. 1.4b Impulse response $s_0(t)$ and corresponding amplitude function $|S_0(f)|$ of a narrowband transducer.

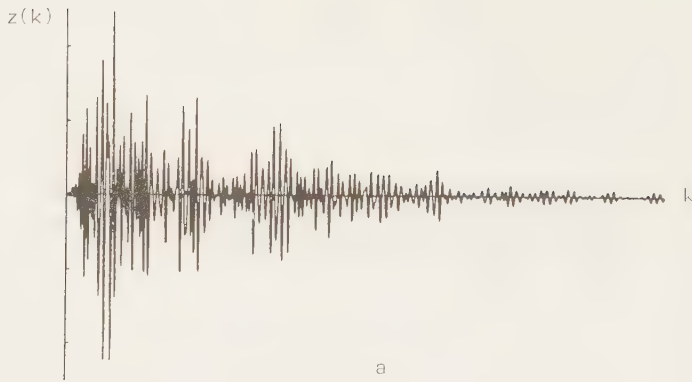


Fig. 1.5 Simulated ultrasonic signal with the echo amplitude given by Fig. 1.2a for
a. the broadband transducer in Fig. 1.3a,
b. the narrowband transducer in Fig. 1.3b.

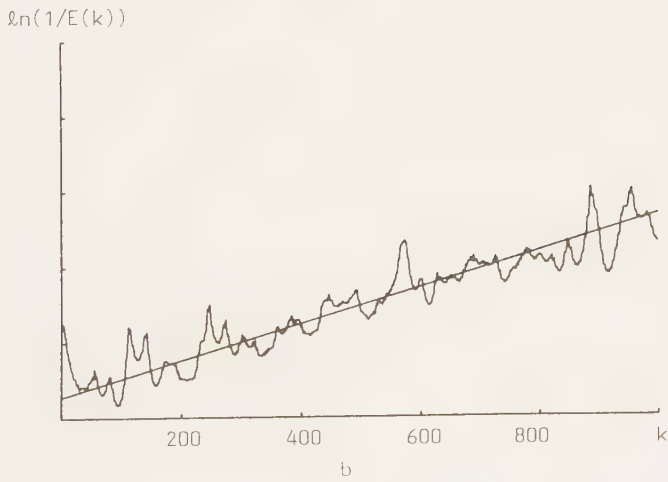
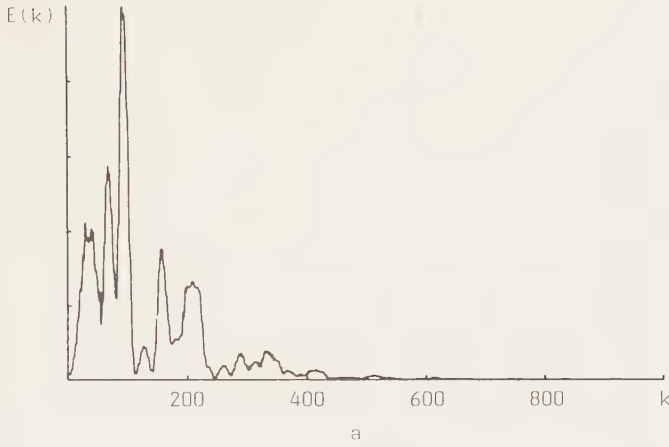


Fig. 1.6a-b Short-time energy $E(k)$ of the ultrasonic signal (Fig. 1.6a) and $\ln(1/E(k))$ with a straight mean-squares line (Fig. 1.6b) for a broadband transducer.

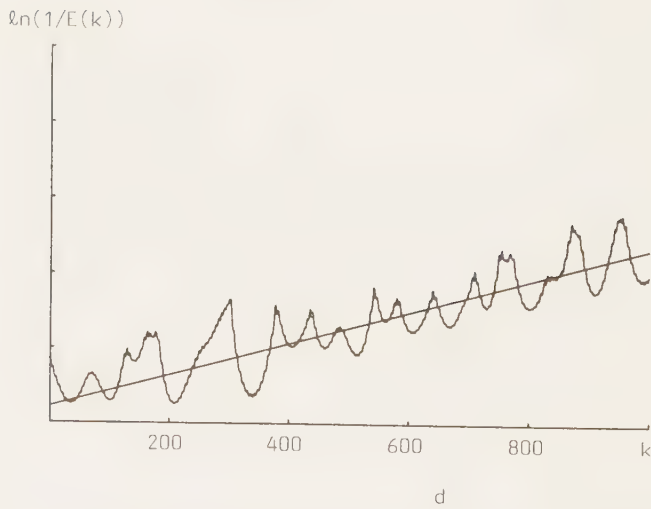
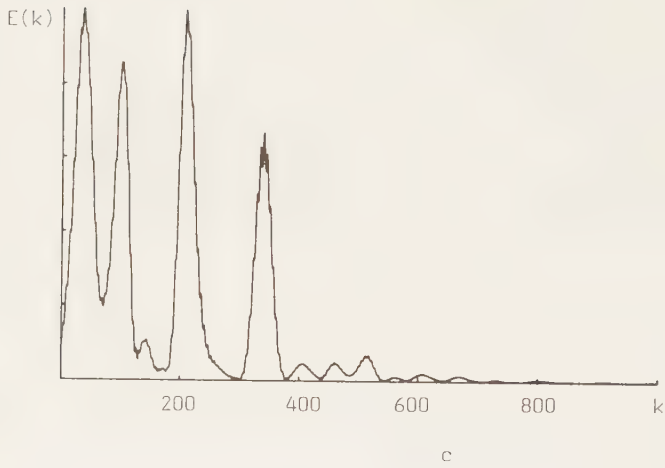


Fig. 1.6c-d Short-time energy $E(k)$ of the ultrasonic signal (Fig. 1.6c) and $\ln(1/E(k))$ with a straight mean-squares line (Fig. 1.6d) for a narrowband transducer.

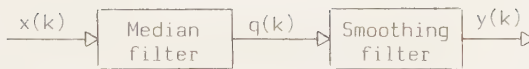
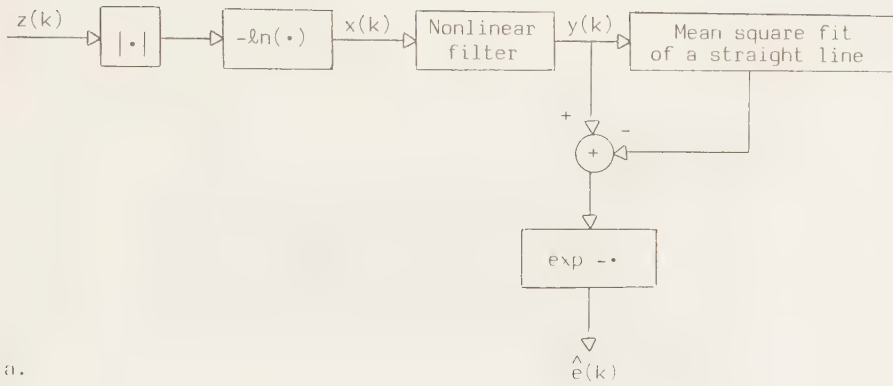


Fig. 1.7 Estimation of the attenuation and the envelope by using a nonlinear filtering method.

a. Block diagram.

b. Structure of the nonlinear filter.

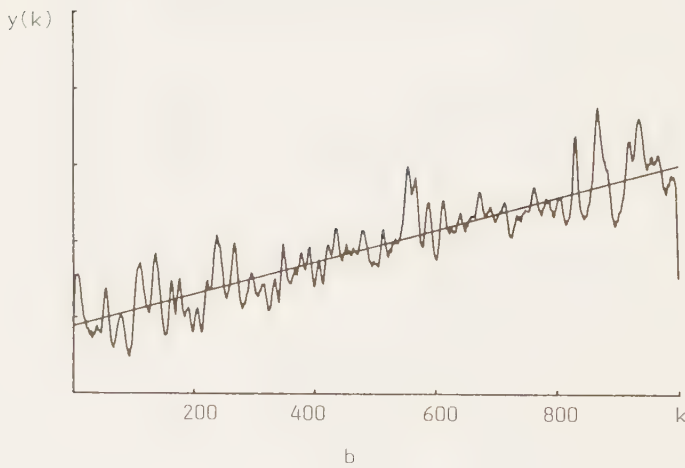
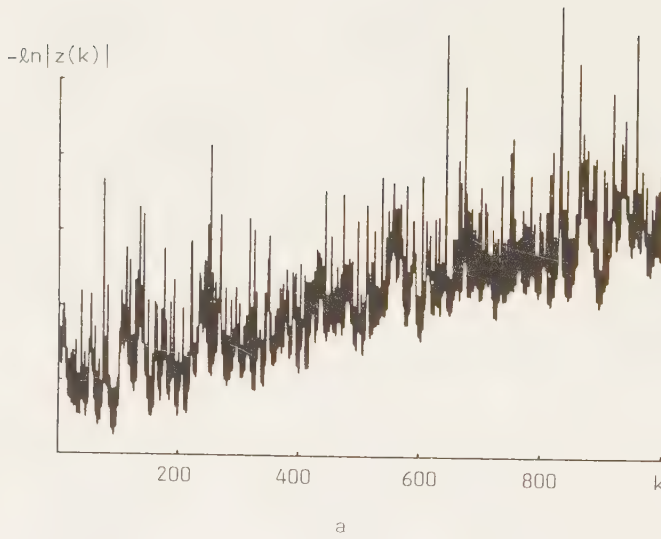


Fig. 1.8a-b $-\ln|z(k)|$, Fig. 1.8a, and nonlinear filtered signal ($K=2$, $L=2$) $y(k)$, Fig. 1.8b, with mean-squares straight line fitted for a broadband transducer.

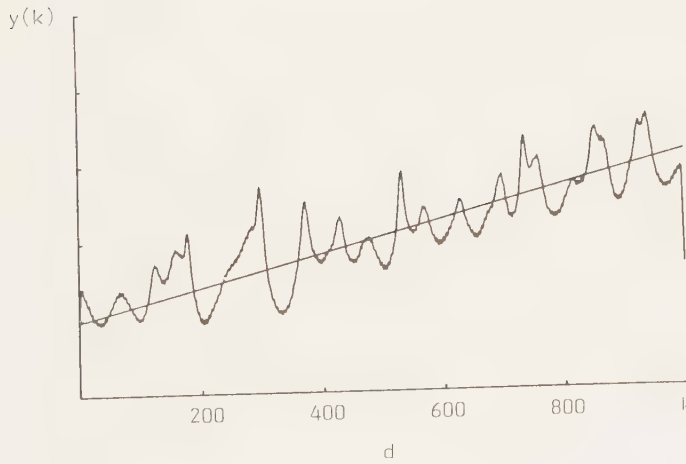
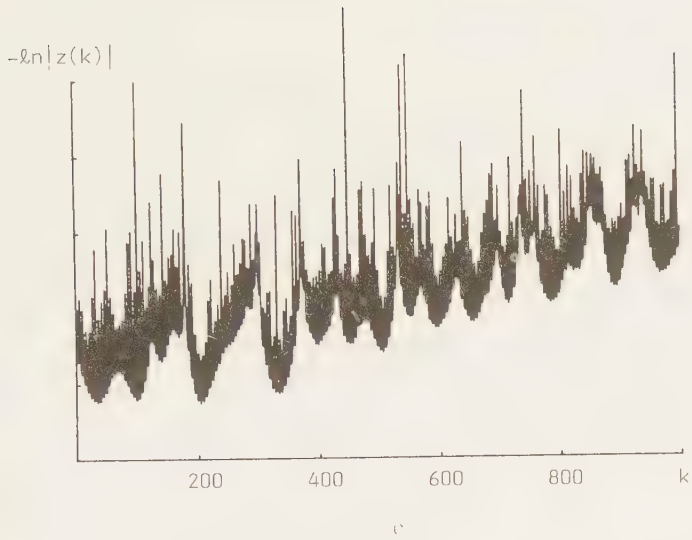


Fig. 1.8c-d $-\ln|z(k)|$, Fig. 1.8c, and nonlinear filtered signal $\hat{y}(k)$, Fig. 1.8d, with mean-squares straight line fitted for a narrowband transducer.

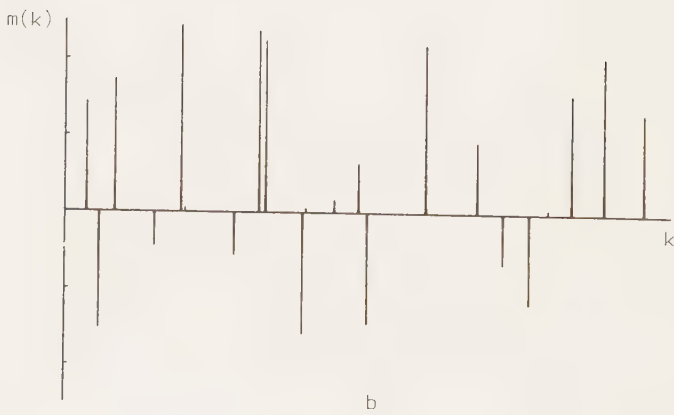
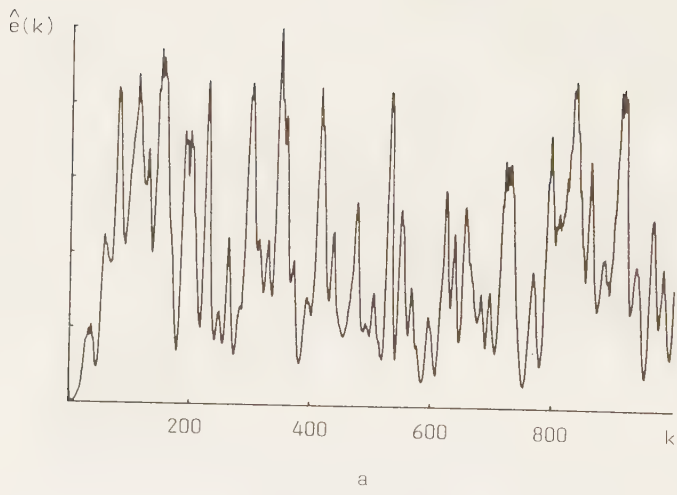
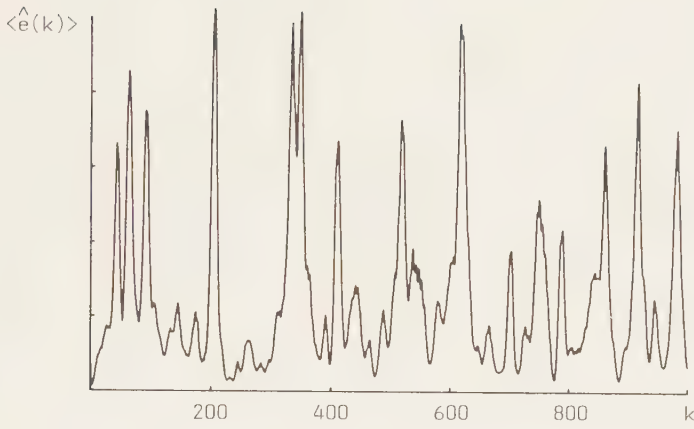
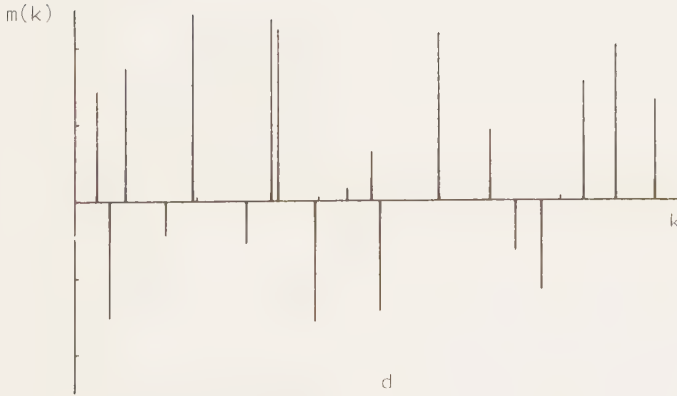


Fig. 1.9 Cont.



c



d

Fig. 1.9 Estimated envelope $\hat{e}(k)$ for a simulated ultrasonic signal with broadband transducer.

- The estimated envelope $\hat{e}(k)$ of a single reflected signal.
- The positions of the specular echoes.
- The envelope $e(k)$ obtained by averaging over 16 reflected signals.
- The positions of the specular echoes repeated for comparison.

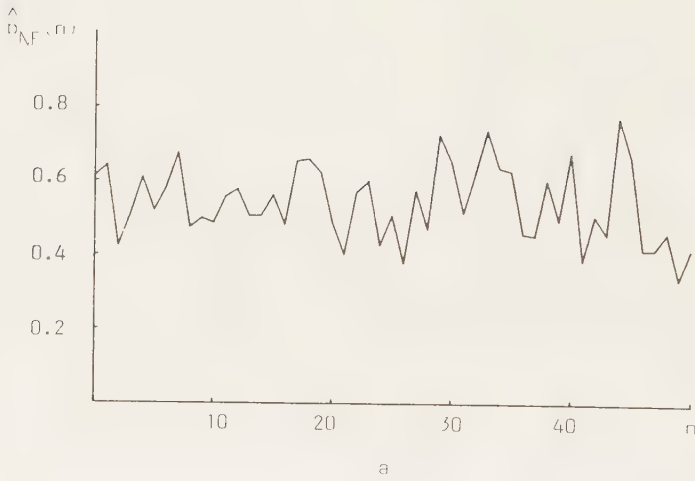


Fig. 1.10a-b Individual estimates $\hat{b}_{NF}$, Fig. 1.10a, and successive averages $\langle \hat{b}_{NF} \rangle$, Fig. 1.10b, for a narrowband transducer. Estimation interval $k = (200-800)$.

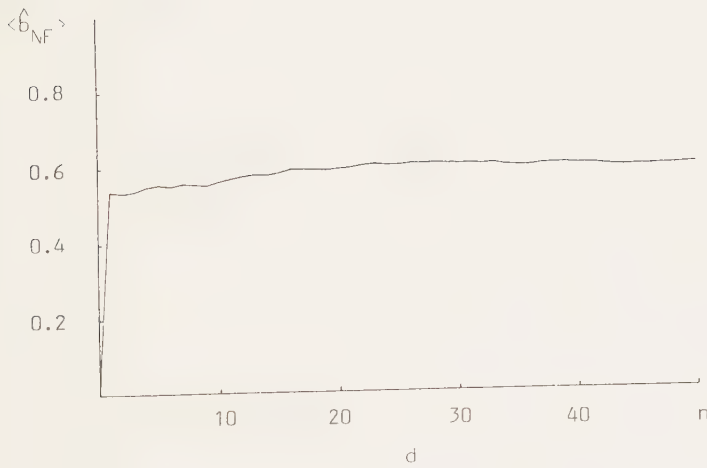
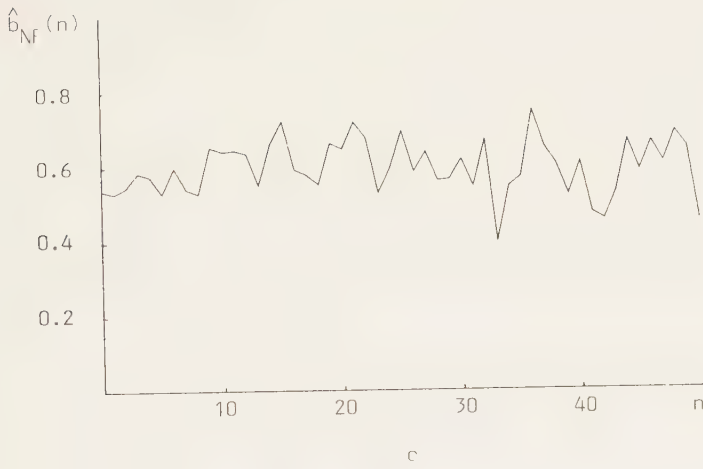


Fig. 1.10c-d Individual estimates $\hat{b}_{NF}$, Fig. 1.10c, and successive averages $\langle \hat{b}_{NF} \rangle$, Fig. 1.10d, for a broadband transducer. Estimation interval $k = (200-800)$.

IMAGING OF VARYING ULTRASONIC ATTENUATION
A COMPARISON OF FREQUENCY AND TIME DOMAIN ESTIMATION METHODS

PART 2

ABSTRACT

Two methods for estimation and imaging of varying ultrasonic attenuation are compared. The first method works entirely in the time domain while the second uses Fourier transforms. The methods are applied both to simulated and real ultrasonic signals obtained from a tissue equivalent. It is shown that the time domain estimation images are superior to those obtained using the frequency domain method.

1. INTRODUCTION

The attenuation of ultrasonic signals has shown to be a promising parameter which provides information on the physiological state of soft tissue. It is desirable not only to measure the average attenuation of a whole organ, but also to measure the attenuation in small parts of an organ. This implies the need for attenuation images showing the ultrasonic attenuation in various parts of an organ. In Part 1, a time domain method for estimation of the attenuation was described. The method uses the decay of the envelope of the ultrasonic signal where the envelope was obtained by means of nonlinear filtering. In this paper, this method is further analyzed and adapted for estimation of varying attenuation in the tissue. The method is then compared to a frequency domain method using the spectral difference between short time spectra (KU-84). Attenuation images are used both from simulated signals and signals obtained from a phantom.

2. TIME DOMAIN ESTIMATION METHOD

A nonlinear time domain estimation method using the log-envelope of the received signal was developed in Part 1. An estimate of the attenuation was obtained by fitting a straight line to the log-envelope in the mean-square sense. Here, the optimum linear estimator of the attenuation from the log-envelope is derived. Furthermore, the method is adapted for estimation of varying attenuation in order to provide attenuation images. The first step in estimating the attenuation is to calculate the log-envelope from the reflected signal. This can be done by means of an ordinary quadrature envelope demodulator and by taking the logarithm of the output. However, a carrier frequency has to be chosen and demodulator low-pass filters have to be designed according to the frequency properties of the transducer. Thus, for any other transducer a new carrier frequency has to be chosen and new lowpass filters must be designed. Furthermore, the computational load using a quadrature demodulator is considerable. In order to avoid the above mentioned drawbacks, the log-envelope is determined according to the block diagram shown in Fig. 2.1. The method requires a modest amount of computations and has the advantage of being independent of the transducer. Furthermore, the method provides a signal that is suitable for estimation of the attenuation. To form the signal $x_i(k)$, the logarithm of the absolute value of a realization $z_i(k)$ of the reflected signal is taken. The input signal $x_i(k)$ to the nonlinear filter is

$$x_i(k) = B(k) - \ln e_i(k) - \ln |\cos(2\pi f_0 k + \phi_i(k))| \quad (2.1)$$

(cf. 1.17)

where

$$B(k) = A(k) + \Gamma(k) r_0^B \quad (2.2)$$

(The notation follows Part 1.) When $\cos[2\pi f_0 k + \phi_i(k)]$ is close to zero, short spikes with very large amplitude appears. Before further processing, these spikes have to be filtered out. The filter function must be of short duration in the time domain, which is achieved by using a nonlinear filter consisting of a median filter followed by a short linear smoothing filter as shown in Fig. 2.1b. The input-output relation of a median filter of length $2K+1$ is given by

$$q_i(k) = \text{Median}_{\ell=-K}^K \{x_i(k+\ell)\} \quad (2.3)$$

Large values of short duration will be eliminated by this filter and the following linear filter of length $2L+1$ smooths the signal. In the investigations below very short filters have been used, some typical values being $K=2-3$ and $L=2-3$.

The output $y_i(k)$ is a noisy realization of

$$B(k) - \ln e_i(k)$$

...:

$$y_i(k) = B(k) - \ln e_i(k) + \epsilon_i(k) = B(k) + v_i(k) + m_i \quad (2.4) \quad (\text{cf. 1.19})$$

Thus $y_i(k)$ is the sum of the unknown signal $B(k)$ that is to be estimated and the noise $v_i(k)+m_i$, where $v_i(k)$ is a realization of a zero-mean random process. To illustrate the properties of the algorithm in Fig. 2.1 the output signal $y_i(k)$ and the "envelope" $\exp\{-y_i(k)\}$ are compared to the corresponding signals obtained from an ordinary quadrature demodulator. It is easily concluded from Fig. 2.2, that a difference between the two methods is that $y_i(k)$ has more rounded-off "spikes" than the corresponding signal from the quadrature demodulator. However, this difference will have a negligible effect on the estimate of $B(k)$. The signal $y_i(k)$ in Eqn. (2.4) is therefore suitable for use in the estimation process. We now face an ordinary communication theory problem: to estimate an unknown signal $B(k)$, that is disturbed by additive noise $v_i(k)$ (IRE-68). The optimal solution in the minimum mean-square sense requires statistical knowledge of the noise $v_i(k)$. Since $y_i(k)$ is obtained by nonlinear processing of the observed signal $z_i(k)$, it is difficult to calculate the exact properties of $v_i(k)$. However, Fig. 2.3 shows that the bandwidth of $v_i(k)$ depends upon the bandwidth of the transducer. Furthermore, we note that the variance of $v_i(k)$ is quite large and thus causes a poor estimate of $B(k)$ independent of which estimator is used. In order to decrease the variance of the noise several realizations $y_i(k)$ are averaged as shown in Fig. 2.1. Thus $B(k)$ is estimated by using

$$y(k) = B(k) + v(k) + m \quad (2.5a)$$

with

$$v(k) = \frac{1}{N} \sum_{i=1}^N v_i(k) \quad (2.5b)$$

The averaging procedure will in addition to decreasing the variance also make the noise approximately Gaussian. In the appendix, the minimum variance estimator is derived assuming constant attenuation in the interval of interest i.e.

$$B(k) = b(f_0)kTv = b \cdot k \quad (2.6)$$

The estimator is expressed in terms of the covariance matrix of the noise $v(k)$, which unfortunately is unknown. However, despite the fact that $v(k)$ in general is coloured, the estimator based on a white noise assumption is found to be useful provided that the studied interval is sufficiently large. Furthermore, this simple estimator is shown to be equivalent to a mean-square fit of a straight line to $y(k)$.

An obvious application of this method for detecting varying attenuation is to estimate the slope inside a moving window of length N , see Fig. 2.4. The slope $\hat{b}$ at a point n is then calculated by minimizing the error

$$\epsilon(n) = \sum_{k=n-N/2}^{n+N/2} [y(k) - \hat{b}(n)k - \hat{m}(n)]^2 \quad (2.7)$$

The estimate $\hat{b}(n)$ of the slope is, see appendix, given by

$$\hat{b}(n) = \frac{\sum_{k=n-N/2}^{n+N/2} (k-n)y(k)}{\sum_{k=n-N/2}^{n+N/2} (k-n)^2} = 12 \frac{\sum_{m=-N/2}^{N/2} my(m+n)}{N(N+1)(N+2)} \quad (2.8)$$

By applying this procedure to several lines, an image can be produced which depicts the variation of the attenuation, i.e. an attenuation image is obtained.

Frequency Domain Method

The frequency domain method is presented in detail in (KU-84) where it is denoted the spectral difference method. This estimation scheme is based on the assumption that the acoustic attenuation increases linearly with frequency ($\beta=1$) i.e.

$$|H(f, x)| = e^{-ax} e^{-\gamma|f|x} \quad (2.9)$$

and estimates the slope γ of this linear dependence. The method is illustrated in Fig. 2.5. In order to obtain an estimate, two intervals (I_0 and I_1) having the relative distance Δx , are chosen. The signals $z_{I_0}(k)$ and $z_{I_1}(k)$ are gated out by means of data windows and their spectra $|Z_{I_0}(f)|$ and $|Z_{I_1}(f)|$ are calculated, Fig. 2.6. The spectral contents of these two signals are approximately given by

$$|Z_{I_0}(f)| \approx |S_0(f)| e^{-ax_0} e^{-\gamma|f|x_0} \quad (2.10a)$$

$$|Z_{I_1}(f)| \approx |S_0(f)| e^{-ax_1} e^{-\gamma|f|x_1} \quad (2.10b)$$

where $S_0(f)$ denotes the two-way impulse response of the transducer. The log-spectral difference is calculated

$$\log|Z_{I_0}(f)| - \log|Z_{I_1}(f)| \approx a\Delta x + \gamma|f|\Delta x \quad (2.11)$$

and an estimate of $\gamma \cdot \Delta x$ is obtained by fitting a straight line to the curve, Fig. 2.7. The slope of the line is thus the estimate of $\gamma \cdot \Delta x$. Finally the estimate of the attenuation coefficient γ is obtained by dividing the estimate with the distance Δx . Similar to the time domain method an attenuation image can be obtained by letting the data windows slide with a fixed relative distance and by applying the estimation scheme to several lines.

3. APPLICATION TO SIMULATED SIGNALS

In order to quantify the performance of the two estimation methods, the true value of the attenuation must be known. Since it is difficult to gain this knowledge from real signals the estimation methods were first applied to simulated signals where all parameter values could be controlled. These simulated signals were generated by means of the model described in Part 1. It is reasonable to assume that the parameters a and γ in Eqn. (1.3) are constant within a healthy organ with homogeneous tissue while a cyst, tumour or infarct area will cause a change of the parameter values. Hence, a simplified tissue model is used. A simulated signal along a line in this "tissue" is shown in Fig. 2.8. The tissue is assumed not to contain any large interfaces such as certain parts of liver tissue (KU-82). In terms of the model this means that $c(i) = w(i)$, i.e. $c(i)$ is a stationary white Gaussian process, see Part 1. The attenuation parameters a and γ are equal to (a_0, γ_0) outside and $(5a_0, 5\gamma_0)$ inside two intervals (k_1, k_2) and (k_3, k_4) where $a_0 = 0.2 \text{ Np/cm}$ and $\gamma_0 = 0.2 \text{ Np/cm MHz}$. The transducer signal $s_0(k)$ was chosen to have broadband character, see Fig. 1.4 and the underlying sampling rate f_s was chosen to 20 MHz.

The two estimation methods are now applied to the simulated signals and the corresponding attenuation estimates are shown in Fig. 2.9 and Fig. 2.10 respectively. For the FDM method rectangular windows of length 128 samples were used with relative distance $\Delta x \approx 1.1 \text{ cm}$ corresponding to 300 samples at the sampling rate $f_s = 20 \text{ MHz}$. The time domain method performs a mean-square fit over 300 samples. In order to reduce the variance the presented estimates were averaged over 8 independent signals. Obviously, both methods are able to detect the two areas of increased attenuation. However, the variance of the estimate obtained by means of the Frequency Domain Method, FDM, is considerably larger than the estimate resulting from the Time Domain Method, TDM, using nonlinear filtering. This result is confirmed in (HE-86) where a statistical study on both simulated and real signals have been performed.

By varying the lengths of the two intervals (k_1, k_2) and (k_3, k_4) for 64 parallel lines, simulated linear array transducer signals from a tissue containing two areas of increased attenuation are obtained, Fig. 2.11. The scattering intensity, σ_c^2 is constant in the whole "tissue". Again, the two methods are applied to the signals from the 64 lines and the results from the two estimation schemes are shown in Fig. 2.12 together with a convention-

al TGC-compensated envelope image (B-scan). The true attenuation image is also included in Fig. 2.12 where dark colour means increased attenuation. We note that the TDM clearly depicts also the second area of increased attenuation, while it hardly is visible in the FDM image. We also note that the second area of increased attenuation is hidden in the "shadow" behind the first one in the conventional envelope image. The larger variance of the FDM image is also evident.

4. MEASUREMENTS ON A TISSUE EQUIVALENT CONTAINING SPECULAR ECHOES

In the preceeding, the estimation methods were applied to model signals. To judge the value of the simulation results, the estimation methods were also tested on real ultrasonic signals which derive from a tissue equivalent (phantom).

The Tissue Equivalent

The phantom is intended to simulate liver tissue containing an area with increased attenuation, corresponding to e.g. a tumour or some local damage. In this study, a phantom consisting of water, oil and nylon wool is used. Water has very low ultrasonic attenuation as opposed to oil, but neither water nor oil has a structure that reflects the sound and gives scattering echoes. Instead, scattering echoes are obtained by mixing the two liquids with nylon wool having a fibre diameter about 0.05mm. The two liquids are separated by a thin rubber bladder which causes specular echoes. The multiple echoes of the nylon wool cause an increased attenuation in both water and oil, resulting in 0.3 and 0.8 neper/cm respectively at 3.5 MHz. In Fig. 2.13 the phantom is illustrated schematically.

The Data Acquisition Equipment

The data acquisition equipment was centered round a HP 5180A waveform recorder, see Fig. 2.14. A voltage peak is generated by the Panametrics pulser/receiver and sent to the transducer giving a sound burst into the medium. The transducer is a 13mm piezoelectric 3.5 MHz transducer with a frequency function according to Fig. 2.15. The reflected sound waves are converted back to a voltage signal by the transducer, amplified by the pulser/receiver and fed through a lowpass filter to the HP 5180A waveform recorder. The waveform recorder samples and converts the signal into digital representation with $f_s = 20$ MHz and 10 bit accuracy. The waveform recorder has only 16k-sample local storage which is not sufficient for an entire recording (see below). Therefore the data is transferred via a HP-IB bus to a buffer memory handled by a Texas 9900 microprocessor. This processor has a serial connection to the NORD 100 host computer, where the signal

processing is performed. Subsequent to this processing the data is sent to a videoprocessor which presents the data as a video image containing 512×512 picture elements with grey scale representation (8 bits).

The tissue equivalent has a depth of about 15cm which means that the number of samples after each pulse must be around 4000 (at a sampling rate of 20 MHz and a mean sound velocity of 1540m/s) if echoes from the top to the bottom are to be sampled. In order to create an image, the transducer is positioned at 64 points along a line at the top of the phantom and a recording is made at each position. Since the estimation methods use averaging over a number of realizations, four parallel images with a relative distance of 1mm are recorded, see Fig. 2.16. The distance between the image planes is chosen in order to make the scattering echoes of the different images independent. In an in vivo tissue this separation is not needed, since there is a natural stochastic variation of the echo structure in time. All in all this means that $4 \times 64 \times 4096 = 1024k$ samples are required for one image.

Estimation Results

Figure 2.17 shows an original RF signal obtained from a single line in an image. Initially some very large echoes occur which originate from the transmitted pulse. The sudden increase in amplitude at the end of the signal derives from the bottom of the tissue equivalent. From this signal it is possible to detect visually an increased attenuation immediately before the middle of the signal. This change could be used diagnostically by a skilled operator. However such a procedure is not reliable. The two estimation methods are now applied to the data received from the phantom. The results averaged over 4 signals are shown in Fig. 2.18. For the FDM the data window lengths were $L=128$ with a relative distance of 300 samples, while the TDM used a mean-square fit within a window of 300 samples. All attenuation values below zero were set to zero. In the TDM estimate the three regions with different attenuation are clearly distinguished. The FDM produces an estimate where it is difficult to find these regions. It is also easily seen that the variance of the FDM estimate is substantially larger than the variance of the TDM estimate. The dips preceeding the increase and succeeding the decrease of the TDM attenuation estimate are caused by the interface echo from the rubber membrane.

Finally, the estimation methods are applied to all the 64 lines giving two-dimensional images, representing the estimated attenuation in the tissue. Figure 2.19 shows such images of the phantom together with a TGC-compensated envelope image. The image is similar to that produced by an ordinary ultrasonic equipment. In the envelope image the region of increased attenuation is seen indirectly by the lower scattering echo intensity, but little information about the size or shape of the region is possible to gain. The TDM estimate shows the region of increased attenuation distinctly and the circular shape is visible as opposed to the FDM image.

5. SUMMARY AND DISCUSSION

In this paper two methods to produce attenuation images are compared. The first method works entirely in the time domain while the second uses Fourier transforms. Applied on simulated signals, both methods produce images that depict the attenuation in the simulated tissue. The simulation results show, furthermore, that the estimates obtained directly in the time domain are more robust than the frequency domain estimates.

This result is further emphasized after testing the methods on real signals from a tissue equivalent. An area of increased attenuation goes undetected in the frequency domain estimates. The same area is clearly detected in the time domain estimation image where even the shape of the area can be seen. Why is the FDM image from the tissue equivalent so poor? The main reasons are probably the large variance of the FDM estimates (HE-86) combined with the large sensitivity to specular echoes. The TDM, however, reduces the effect of specular echoes and the estimates have considerably lower variance. Another difference is that the FDM only detects the frequency dependent part of the attenuation as opposed to the TDM where both frequency dependent and frequency independent attenuation contributes to the estimates.

The variance can, of course, be reduced by averaging over many more signals as proposed in (KU-84). The computational load for the FDM is, however, already considerable without performing extensive averaging, due to the numerous Fourier transforms. (Comparing the two methods the TDM requires less computation time for the whole 64-line image than the time needed for an estimation along a single line using the FDM.) If the aim is to produce attenuation images on-line, the TDM is more attractive combining short computation time with robust estimates of the attenuation.

APPENDIX: Derivation of the Linear Minimum Variance Estimator

The problem is to estimate the attenuation in a given interval $k \in [0, \dots, N]$. Thereby $B(k)$ is assumed to be a linear function, i.e., the attenuation of the tissue is assumed to be constant in the interval, so that the preprocessed signal takes the form

$$y(k) = b \cdot k + v(k) + m \quad (\text{A.1})$$

The noise $v(k)$ is a zero-mean random process with the covariance function

$$E\{v(k+l)v(k)\} = \sigma_v^2 \delta(l) \quad (\text{A.2})$$

No assumptions about the distribution of $v(k)$ and b are made.

To estimate the slope b a linear estimator is used i.e.

$$\hat{b} = \hat{b}(N) = \sum_{n=0}^N u(n) y(n) \quad (\text{A.3})$$

where the weights $\{u(n); n = 0, \dots, N\}$ are to be determined. Alternatively the estimate can be interpreted in terms of linear filtering. In this case $y(k)$ is the input signal to a linear filter with the impulse response $u'(k)$ $0 \leq k \leq N$. The output signal is

$$\hat{b}(k) = \sum_{n=0}^k u'(k-n)y(n) \quad k \leq N \quad (\text{A.4})$$

The value of $\hat{b}(k)$ at $k=N$ is used as the estimate of b , i.e.

$$\hat{b} = \hat{b}(N) = \sum_{n=0}^N u'(N-n)y(n) \quad (\text{A.5})$$

where

$$u(n) = u'(N-n) \quad (\text{A.6})$$

Inserting Eqn. (A.1) into Eqn. (A.3) yields

$$\hat{b} = b \sum_{n=0}^N nu(n) + \sum_{n=0}^N v(n)u(n) + m \sum_{n=0}^N u(n) \quad (\text{A.7})$$

In common vector notation Eqn. (A.7) will be

$$\hat{b} = \underline{b} \underline{u}^T \cdot \underline{s}_1 + \underline{u}^T \cdot \underline{v} + m \cdot \underline{u}^T \cdot \underline{s}_2 \quad (\text{A.8})$$

where $\underline{u}^T = (u(0), u(1), \dots, u(n))$; $\underline{v}^T = (v(0), v(1), \dots, v(N))$; $\underline{s}_1^T = (0, 1, \dots, N)$; $\underline{s}_2^T = (1, 1, \dots, 1)$. To obtain an unbiased estimate the impulse response $u(k)$ must be constrained by

$$(1) \quad \underline{u}^T \underline{s}_1 - 1 = 0 \quad (\text{A.9a})$$

$$(2) \quad \underline{u}^T \underline{s}_2 = 0 \quad (\text{A.9b})$$

With these constraints the estimation error will be

$$\hat{b} - b = \underline{u}^T \underline{v} \quad (\text{A.10})$$

and its variance

$$V\{\hat{b} - b\} = E\{\underline{u}^T \underline{v} \underline{v}^* \underline{u}\} = \underline{u}^T \underline{R} \underline{u} \quad (\text{A.11})$$

The element r_{ij} of the covariance matrix $\underline{R} = E\{\underline{v}^* \underline{v}\}$ is $r_{ij} = \sigma_v^2 \rho(|i-j|)$. The weights $\underline{u}$ are now determined to minimize the variance fulfilling the constraints in Eqn. (A.9). The optimum weights are obtained by means of Lagrangian multipliers, i.e. the function

$$F = \underline{u}^T \underline{R} \underline{u} + 2\lambda_1 [\underline{u}^T \underline{s}_1 - 1] + 2\lambda_2 \underline{u}^T \underline{s}_2 \quad (\text{A.12})$$

is studied. $\underline{u}$, λ_1 and λ_2 are determined by

$$\underline{\nabla} F = 2 \underline{R} \underline{u} + 2\lambda_1 \underline{s}_1 + 2\lambda_2 \underline{s}_2 = 0 \quad (\text{A.13})$$

and the constraints. Eqn. (A.13) yields

$$\underline{u}_{\text{opt}} = -\underline{R}^{-1} [\lambda_1 \underline{s}_1 + \lambda_2 \underline{s}_2] \quad (\text{A.14})$$

Putting this expression of $\underline{u}_{\text{opt}}$ into the two constraints, two equations to determine λ_1 and λ_2 are yielded

$$[\lambda_1 \underline{s}_1^T + \lambda_2 \underline{s}_2^T] \tilde{R}^{-1} \underline{s}_1 + 1 = 0 \quad \text{A.15a}$$

$$[\lambda_1 \underline{s}_1^T + \lambda_2 \underline{s}_2^T] \tilde{R}^{-1} \underline{s}_2 = 0 \quad \text{A.15b}$$

The solution to these equations is given by

$$\lambda_1 = \frac{-\underline{s}_2^T \tilde{R}^{-1} \underline{s}_2}{\underline{s}_1^T \tilde{R}^{-1} \underline{s}_1 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2 + \underline{s}_1^T \tilde{R}^{-1} \underline{s}_2 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2} \quad \text{A.16}$$

$$\lambda_2 = \frac{\underline{s}_1^T \tilde{R}^{-1} \underline{s}_2}{\underline{s}_1^T \tilde{R}^{-1} \underline{s}_1 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2 + \underline{s}_1^T \tilde{R}^{-1} \underline{s}_2 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2} \quad \text{(A.16b)}$$

$$\underline{u}_{\text{opt}} = \frac{\tilde{R}^{-1} [\underline{s}_1 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2 - \underline{s}_2 \underline{s}_1^T \tilde{R}^{-1} \underline{s}_2]}{\underline{s}_1^T \tilde{R}^{-1} \underline{s}_1 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2 + \underline{s}_1^T \tilde{R}^{-1} \underline{s}_2 \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2} \quad \text{(A.16c)}$$

The expressions can be simplified by observing that $\tilde{R}$ (and $\tilde{R}^{-1}$) is unaltered by a mirroring in its center-point (R is Toeplitz and symmetric) i.e.

$$r_{ij} = r_{(N+1-i) (N+1-j)} \quad \text{(A.17)}$$

Using this property we obtain

$$\underline{s}_1^T \tilde{R}^{-1} \underline{s}_2 = \frac{N}{2} \underline{s}_2^T \tilde{R}^{-1} \underline{s}_2 \quad \text{(A.18)}$$

Introducing $\underline{s} = \underline{s}_1 - \frac{N}{2} \underline{s}_2$, Eqn. (A.16) can be written as

$$\lambda_1 = - \frac{1}{\underline{s}^T \tilde{R}^{-1} \underline{s}} \quad \text{(A.19a)}$$

$$\lambda_2 = - \frac{N}{2} \lambda_1 \quad \text{(A.19b)}$$

$$\underline{u}_{\text{opt}} = \frac{\tilde{R}^{-1} \underline{s}}{\underline{s}^T \tilde{R}^{-1} \underline{s}} \quad \text{(A.19c)}$$

The estimate is

$$\hat{b} = y^T \underline{u}_{\text{opt}} = \frac{\underline{s}^T \underline{\tilde{R}}^{-1} \underline{s}}{\underline{s}^T \underline{\tilde{R}}^{-1} \underline{s}} \quad (\text{A.20})$$

The corresponding variance is given by

$$\begin{aligned} V[\hat{b}-b] &= \underline{u}^T \underline{\tilde{R}} \underline{u} = \\ &= \frac{1}{2} \underline{u}^T \underline{\nabla F} - \lambda_1 \underline{u}^T \underline{s}_1 - \lambda_2 \underline{u}^T \underline{s}_2 = \\ &= 0 - \lambda_1 - 0 = -\lambda_1 \end{aligned} \quad (\text{A.21})$$

Using Eqn. (A.19a), the variance can be written

$$V[\hat{b}-b] = \frac{1}{\underline{s}^T \underline{\tilde{R}}^{-1} \underline{s}} \quad (\text{A.22})$$

Example 1: White noise assumption; optimal solution.

If the noise $v(k)$ is white with the covariance matrix $R = \sigma_v^2 I$, where I is the unit matrix, we obtain

$$\underline{u}_{\text{opt}} = \frac{I \cdot \underline{s}}{\sum_{n=0}^N \left(n - \frac{N}{2} \right)^2} \quad (\text{A.23})$$

and the estimate is given by

$$\hat{b} = \frac{\sum_{n=0}^N \left(n - \frac{N}{2} \right) y(n)}{\sum_{n=0}^N \left(n - \frac{N}{2} \right)^2} \quad (\text{A.24})$$

This is the same estimate that is obtained by fitting a straight line $\hat{b}k + \hat{m}$ in the mean-square sense to the signal $y(k)$.

Finally, the variance of the estimation error is

$$V[\hat{b}-b] = \frac{12\sigma_v^2}{N(N+1)(N+2)} \quad (\text{A.25})$$

For large values of N the variance is proportional to $1/N^3$.

Example 2: First order lowpass noise; optimal solution.

In the general case we have to determine R^{-1} to obtain the minimum variance estimator. For typical values of N (250-350) this leads to considerable numerical problems. Further from Fig. 2.3 it is obvious that the noise $v(k)$ has more lowpass character using a narrowband transducer than for a broadbanded one. Hence, we choose to study the minimum variance estimator when the noise $v(k)$ is assumed to be coloured with covariance function

$$r_v(l) = E[v(k+l)v(k)] = \sigma_v^2 a^{|l|} \quad 0 \leq a \leq 1 \quad (\text{A.26})$$

i.e. the noise is obtained by feeding a white sequence through a simple first order lowpass filter. Using this simple colouring it is possible to determine R^{-1} analytically and thereby avoiding the numerical calculation of $\tilde{R}^{-1}$. The parameter "a" controls the lowpass character of the noise $v(k)$ and we observe that white noise is obtained as a special case with $a=0$. Assuming the covariance function $r_v(l)$ as above we get

$$\tilde{R} = \sigma_v^2 \begin{pmatrix} 1 & a & a^2 & \dots & a^N \\ a & 1 & a & \dots & a^{N-1} \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & \\ \vdots & & & & 1 \end{pmatrix} \quad (\text{A.27})$$

The inverse $\tilde{R}^{-1}$ is given by

$$\tilde{R}^{-1} = \frac{1}{\sigma_v^2} \cdot \frac{1}{1-a^2} \begin{pmatrix} 1 & -a & & & & \\ -a & 1+a^2 & -a & & & 0 \\ & -a & 1+a^2 & & & \\ & & & \ddots & & \\ 0 & & & & 1+a^{N-2} & -a \\ & & & & -a & 1 \end{pmatrix} \quad (A.28)$$

and putting this expression into Eqn. (A.19c) and (A.22) yields

$$u_{\text{opt}}^{(k)} = \begin{cases} \frac{-\lambda_1 [1-a]^2}{\sigma_v^2 [1-a^2]} \cdot \left[k - \frac{N}{2} \right] & 1 \leq k \leq N-1 \\ -u(0) = u(N) = \frac{-\lambda_1 \left[\frac{N}{2} (1-a) + a \right]}{\sigma_v^2 [1-a^2]} \end{cases} \quad (A.29)$$

where

$$-\lambda_1 = \frac{\sigma_v^2 [1-a^2] \cdot 12}{N[N^2(1-a)^2 + N(3-3a)^2 + 2 + 8a + 2a^2]} \quad (A.30)$$

The estimate of the slope is given by

$$\hat{b} = \sum_{n=0}^N u(k)y(k) \quad (A.31)$$

and the error variance is

$$V[\hat{b}-b] = -\lambda_1 \quad (A.32)$$

We observe that by letting $a \rightarrow 0$ the same results as in Example 1 are obtained. In Fig. 2.20 the weights $u(k)$ for the minimum variance estimator are plotted assuming $N=30$ and $a=0.5$ together with the weights for the least-squares estimator ($a=0$). We observe that the minimum variance estimator weighs $y(0)$ and $y(N)$ more than the estimator based on the white noise assumption.

Example 3: First order coloured lowpass noise; white noise estimator.

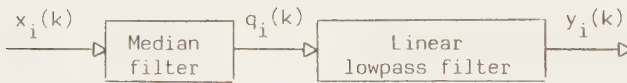
What happens if the estimator for $a=0$ is used on coloured noise? This procedure is equivalent to fitting a least-squares straight line to the curve (disregarding the fact that the noise is coloured). Since Eqn. (A.9) is still satisfied an unbiased estimate of the slope is obviously obtained, but how large is the increase in variance? Putting $u(k)$ from Eqn. (A.23) into Eqn. (A.11) yields

$$v[\hat{b}-b] = \sigma_v^2 \left[\frac{12}{N(N+1)(N+2)} + \frac{24}{(N(N+1)(N+2))^2} \sum_{\ell=1}^N a^{\ell(N-\ell+1)} \left(N^2 - 2N(\ell-1) - 2\ell(\ell+1) \right) \right] \quad (\text{A.33})$$

In Figs. 2.21 a and 2.21b this variance is compared to the variance obtained by using the filter matched to the white noise case. The figures show that for large values of N the simpler estimator ($a=0$) can be used at the cost of a small increase in variance as long as a is not close to 1. This will be the case if the transducer is not extremely narrowband.



a)



b)

Fig. 2.1 a) Block diagram of the calculation of the averaged log-envelope $y(k)$.

b) The nonlinear filter.

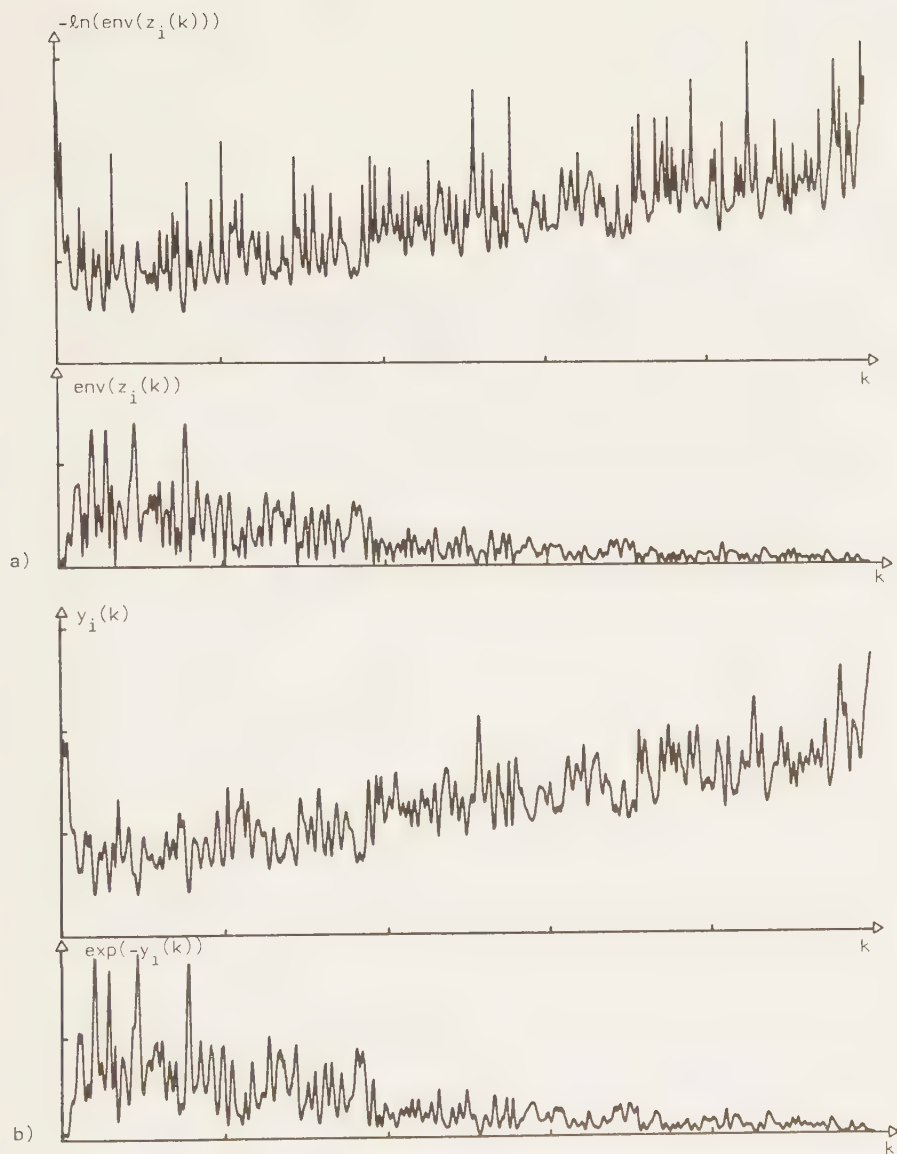


Fig. 2.2. "log-envelope" and "envelope" of received signal for broadband transducer using

- a) Quadrature demodulator
- b) Nonlinear filtering.

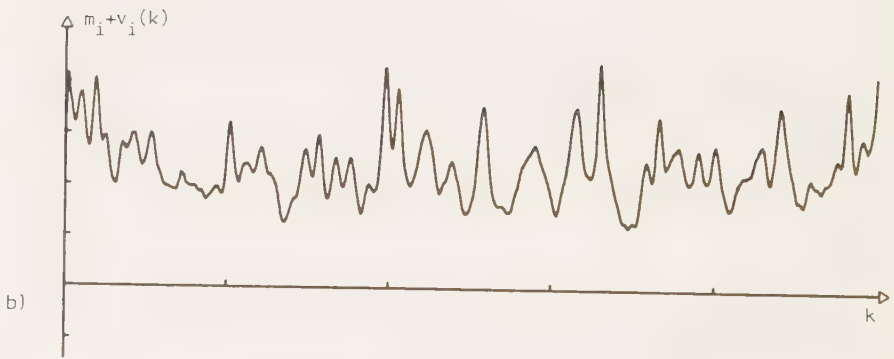
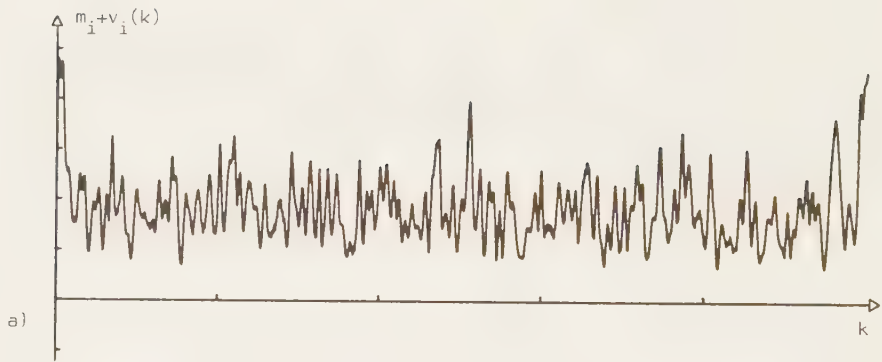


Fig. 2.3 Typical realizations of the noise $m_i + v_i(k)$ for
a) Broadband transducer
b) Narrowband transducer.

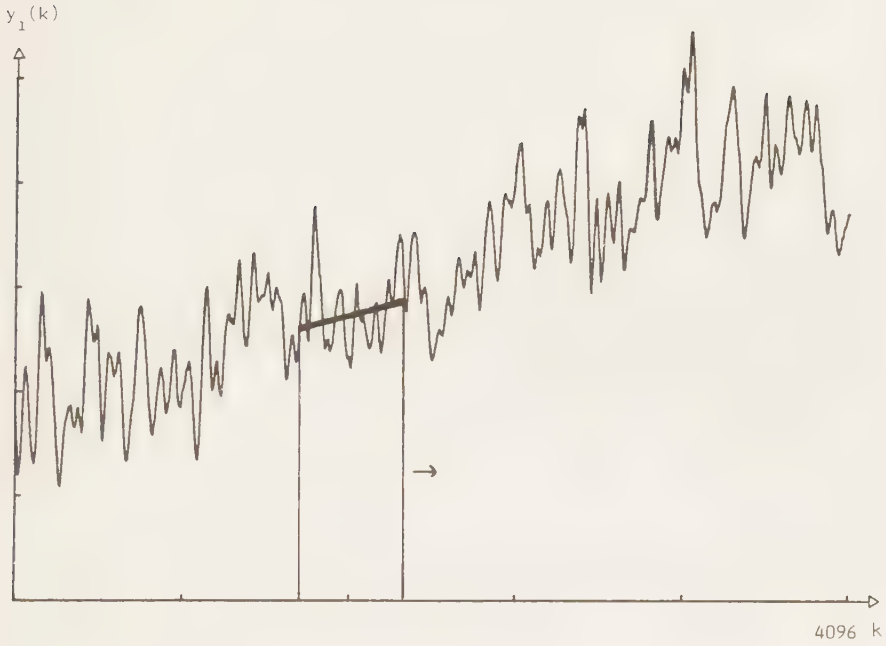


Fig. 2.4 The log-envelope $y_i(k)$ with a straight line fitted within a sliding window. Averaging is obviously needed for variance reduction of the slope estimates.

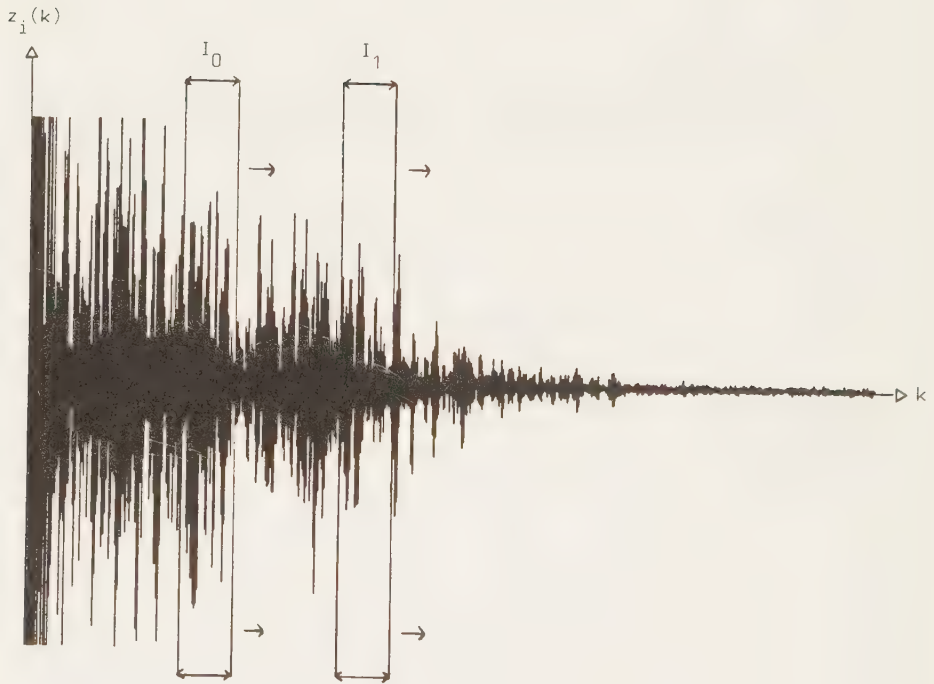


Fig. 2.5 A received RF ultrasonic signal with sliding frequency domain estimation intervals.

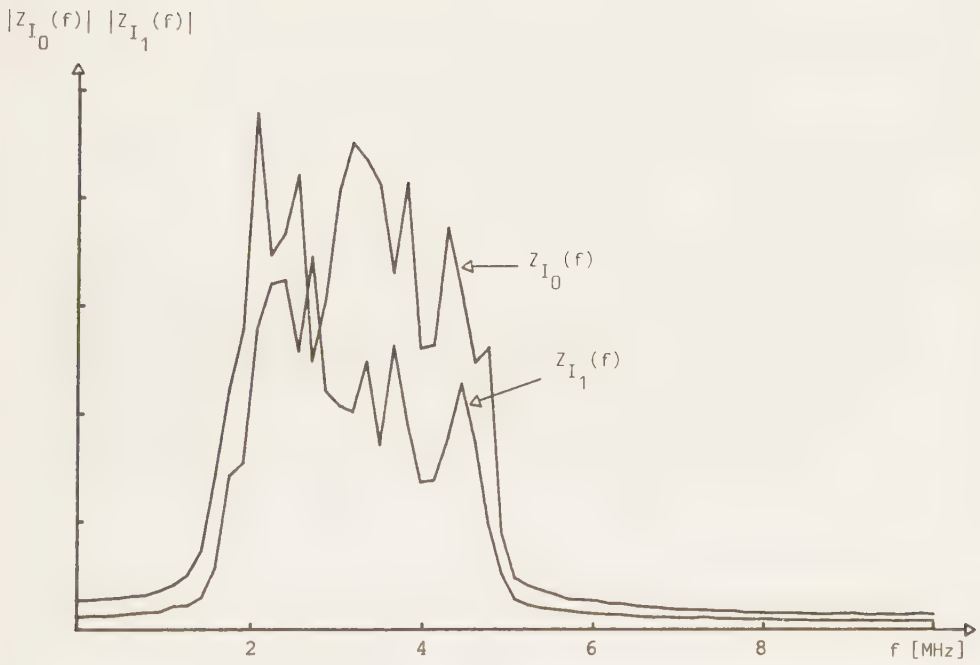


Fig. 2.6 The spectra of the extracted signals from the two intervals I_0 and I_1 .

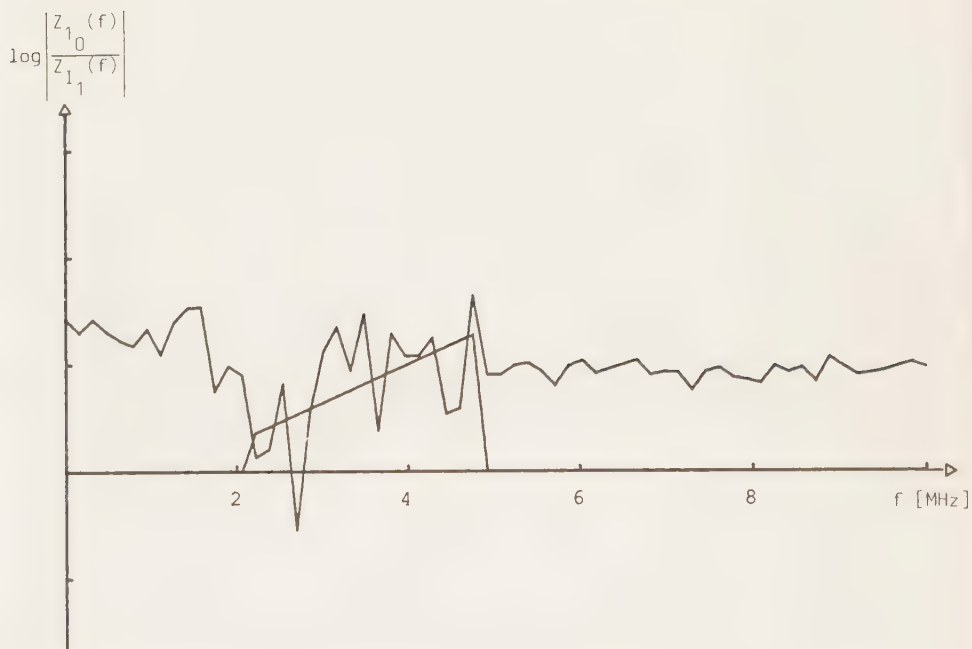


Fig. 2.7 The spectral difference of the two extracted signals with straight line fitted within the frequency range of the transducer.



Fig. 2.8 A simulated signal with two regions of increased attenuation (k_1, k_2) , (k_3, k_4) .

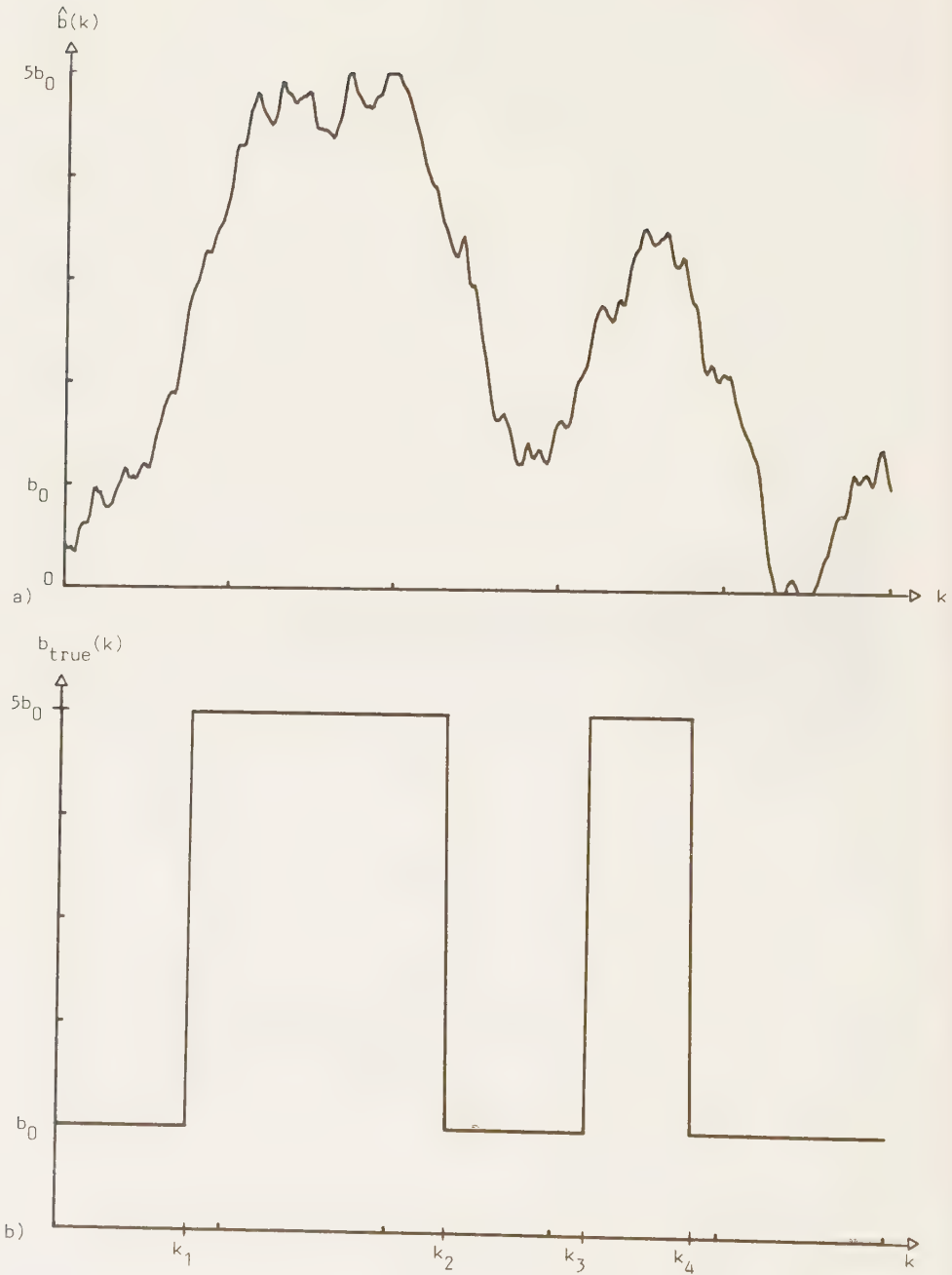


Fig. 2.9 Estimated slope of the attenuation coefficient in the simulated tissue with TDM (a). True values are shown in (b).

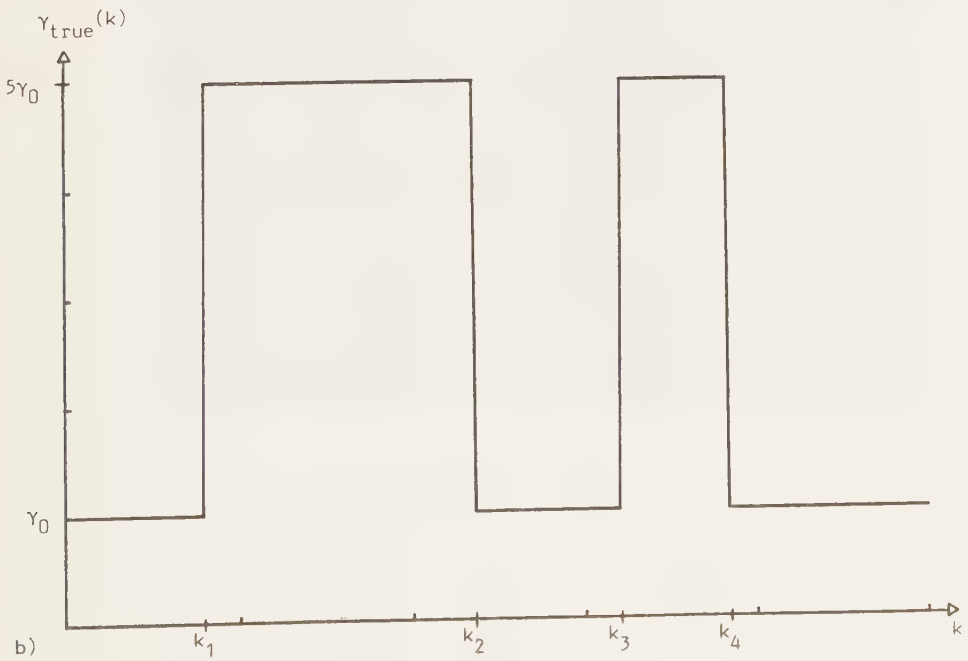
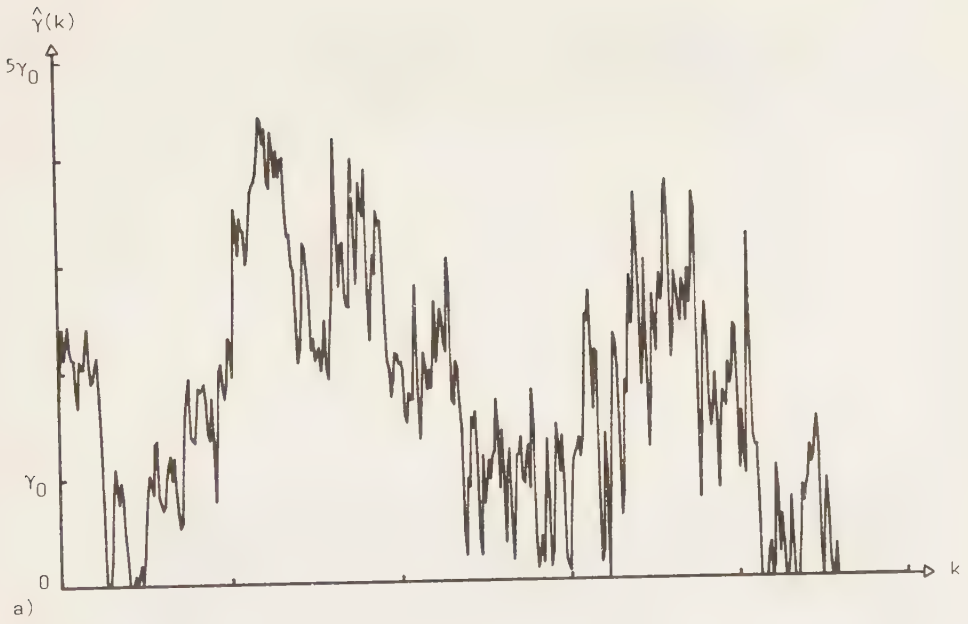


Fig. 2.10 Estimated attenuation coefficient in the simulated tissue with FDM (a). True values are shown in (b).

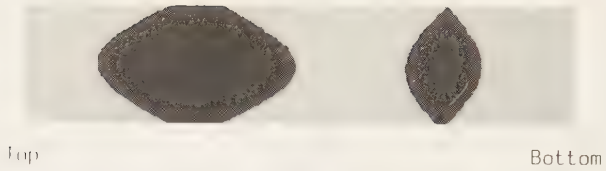


Fig. 2.11 Attenuation coefficient of simulated tissue.
Dark colour means higher attenuation.

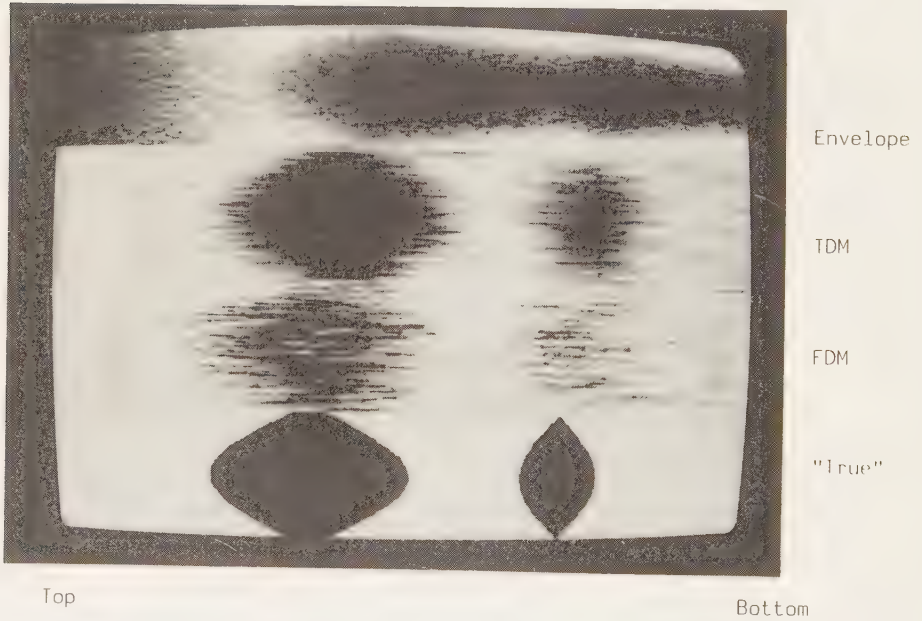


Fig. 2.12 Estimation results of the two methods together with a conventional TGC-compensated image. The true attenuation image is included to enable comparison. Estimates are averages over 8 signals.

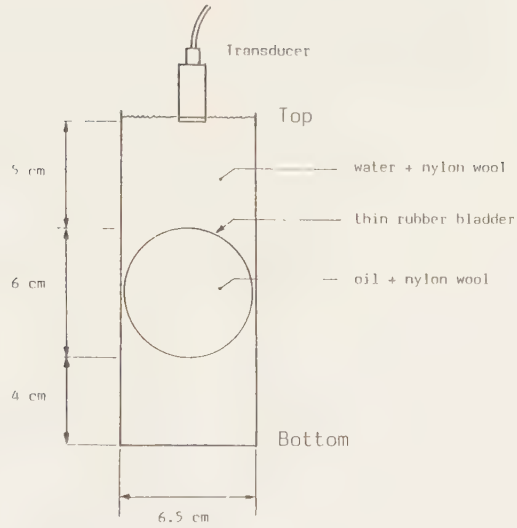


Fig. 2.13 Schematic description of the phantom.

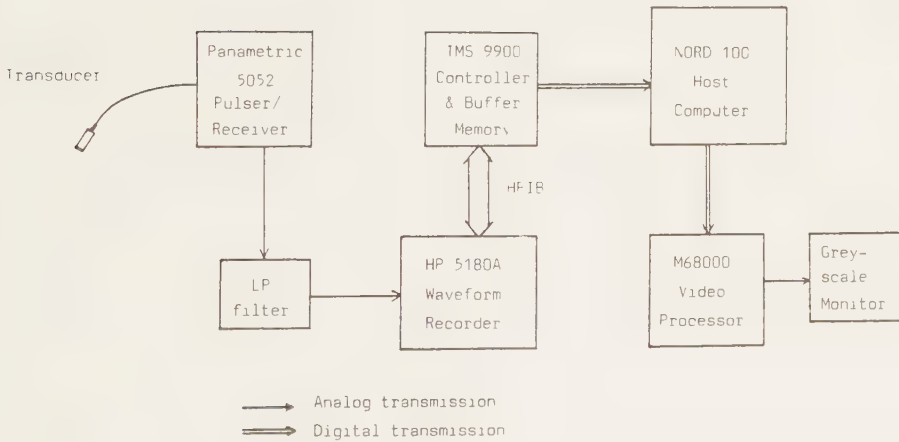


Fig. 2.14 Block diagram of the data acquisition equipment.

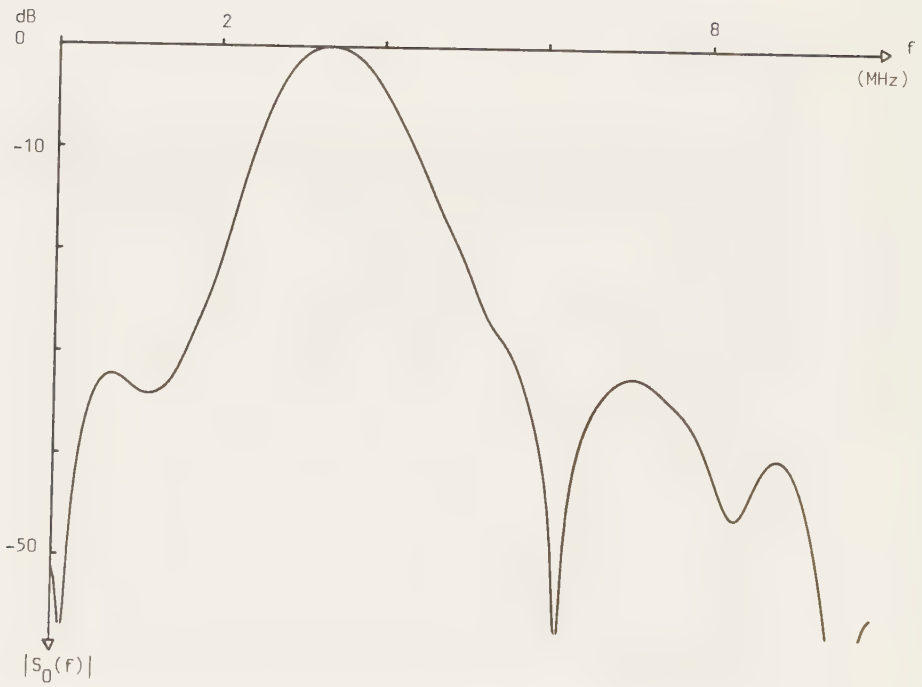


Fig. 2.15 The frequency function $|S_0(f)|$ for the 3.5MHz transducer.

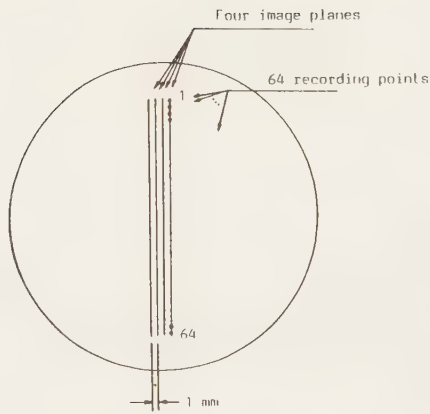


Fig. 2.16 Top view of phantom with recording points marked.

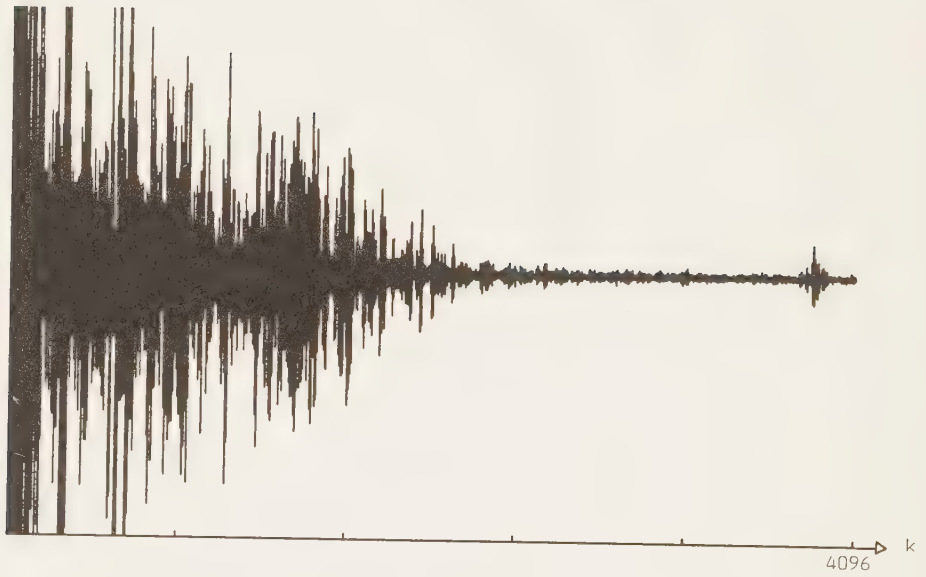


Fig. 2.17 A recorded RF-signal from the phantom.

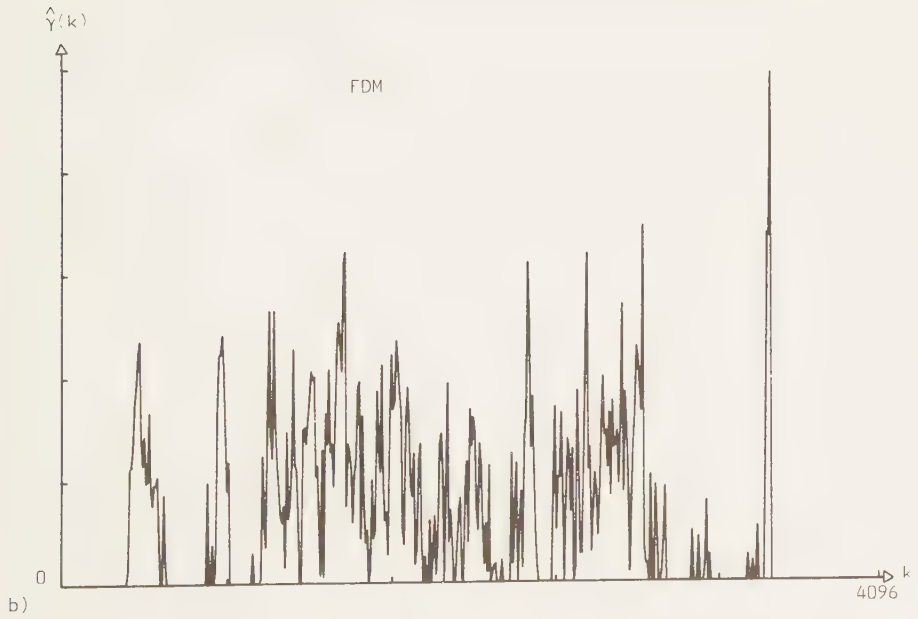
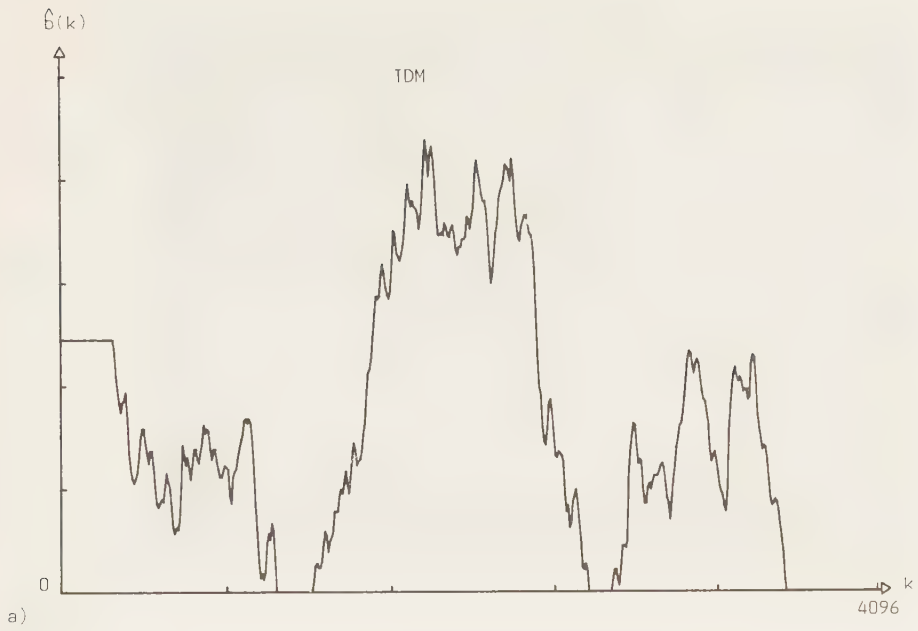


FIG. 2.18 Estimated attenuation along a line in the tissue equivalent according to TDM (a) and FDM (b).

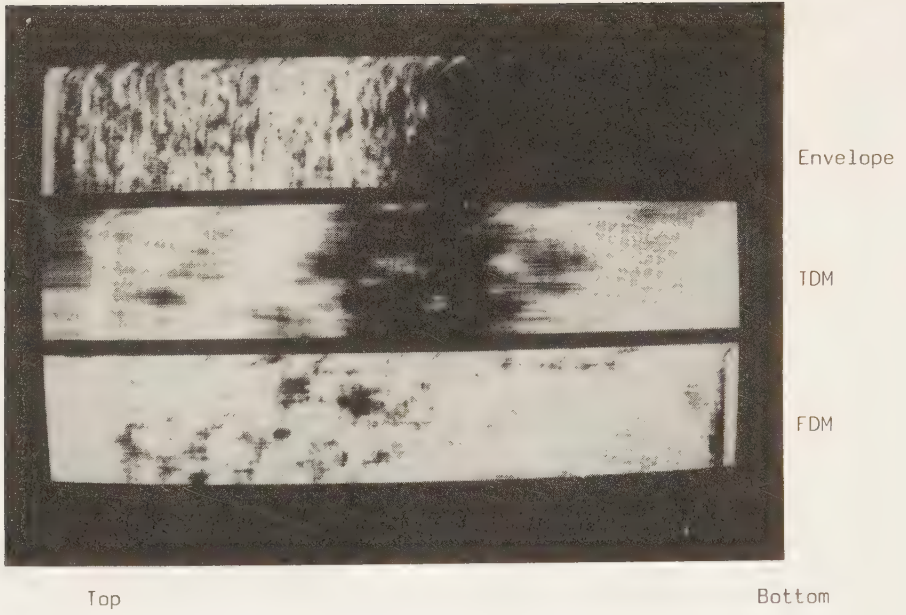


Fig. 2.19 Estimated attenuation images of the tissue equivalent.

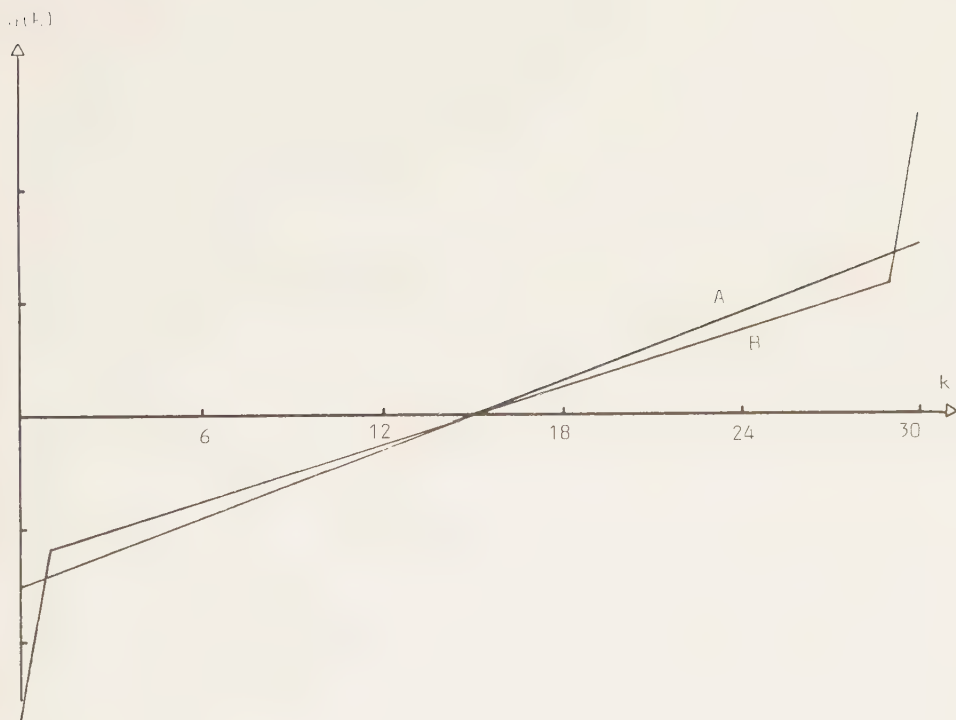


Fig. 2.20. The linear minimum variance weights $u(k)$ for white noise (A) and coloured noise (B) ($a=0.0$ and 0.5 resp.). $N=30$.

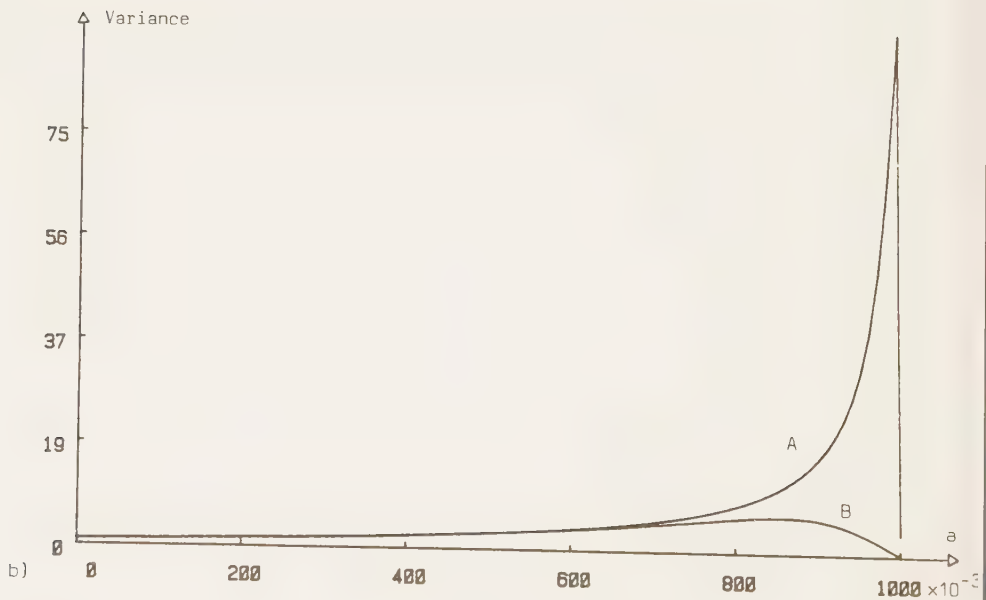
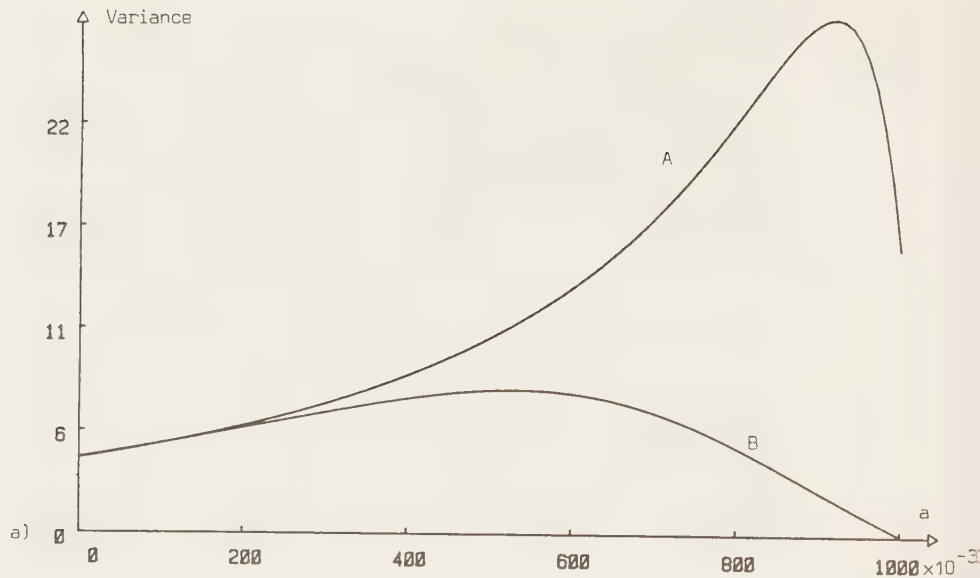


Fig. 2.21 Variances obtained from the least-squares estimator (A) and the linear minimum variance estimator matched to coloured noise input (B) ($r_v(l) = a^{|l|}$) for

a) $N=30$

b) $N=500$



FREQUENCY DEPENDENT COMPENSATION OF ULTRASONIC SIGNALS

PART 3



ABSTRACT

The resolution in ultrasonic images decreases with depth when using traditional Time Gain Compensation (TGC) of the received signal. In order to reduce this effect both frequency and depth-dependent compensation is introduced. A time varying digital filtering method is presented for this purpose which is based on Wiener filtering. Images obtained from a simulated tissue equivalent are presented and the frequency compensated images are compared to conventional TGC-compensated images. It is shown that the improvement in resolution using frequency dependent compensation is considerable when the signal-to-noise ratio in the received signal is high.

1. INTRODUCTION

The attenuation is the most dominant feature of ultrasonic signals and is in itself an interesting parameter for tissue characterization. When analyzing B-scan images, however, the focus of interest is mainly on the anatomy and the texture pattern while the attenuation is of less interest. This is also the case when discrimination methods based on other features, such as A-mode amplitude histograms, are used (JO-79). Although the attenuation is not of primary interest most ultrasonic equipments compensate for the influence of the attenuation before the image is presented and further analysis is done. Such a compensation is traditionally performed by a TGC-circuit (Time Gain Compensation) where the signal is increasingly amplified with depth. The TGC is frequency independent, i.e. all frequencies are equally amplified. The amount of TGC is determined by the operator who adjusts the TGC in order to obtain roughly the same brightness over the whole B-scan image. The attenuation is described mainly by the frequency transmission properties of the tissue. It is a well-known fact that higher frequencies are attenuated more than lower ones in a tissue, a difference which is progressively accentuated with depth. An expression covering the attenuation properties for a wide class of soft tissue was given in Part 1, Eqn. (1.6b). Frequency dependent attenuation implies that echoes from large depths contain essentially the lower frequencies of the exciting pulse (KU-79, KU-81). The frequency dependent attenuation in combination with frequency independent compensation (TGC) results in a gradual loss of the higher frequencies with depth. This loss causes the resolution in the image to deteriorate with depth since the resolution is intimately related to the higher frequencies in a signal. The sharpness of contours deteriorates and the background texture is increasingly distorted. In order to avoid these effects, higher frequencies must be progressively more amplified than lower ones. This procedure can be performed by time-varying, frequency dependent processing of the received signal.

In this paper such a time-varying processor is presented consisting of a sequence of filters, each of which is matched to compensate for the attenuation at a certain depth in the tissue. The filters in the sequence are defined by a bank of bandpass filters. Each filter in the sequence is obtained by different weighting of the bandpass filters. Since only the weighting coefficients differ from filter to filter in the sequence, the processor can alternatively be interpreted as a single time-varying filter. The weighting coefficients are chosen in such a way that the filters approximate a sequen-

ce of Wiener filters. By taking advantage of the bandpass nature of the reflected ultrasonic signal, only a small number of coefficients are required to obtain a good restoration of the ultrasonic image.

2. FREQUENCY DEPENDENT COMPENSATION

In Part 1, a model for simulation of ultrasonic signals with known properties was presented. Signals obtained by that model are useful for developing different signal processing schemes in order to compensate for the attenuation. When compensating weak signals (deep in the tissue) measurement noise must be taken into consideration. Hence, it is incorporated in the model by modifying Eqn. (1.5),

$$x(k) = \sum_i c(i) s_i(k-i) + n(k) \quad (3.1)$$

where $n(k)$ is assumed to be additive white Gaussian noise with variance σ_n^2 . We will later see that the noise level sets a limit to the amount of frequency compensation introduced. If the frequency dependent attenuation is exaggerated the noise will first destroy the background texture deep in the tissue where the signal-to-noise ratio is poor due to the depth dependent attenuation and the fact that the background texture originates from weak scattering echoes. By letting the signal-to-noise ratio between the weak scattering echoes and the noise determine the limit for the compensation, we ensure that the compensation is not exaggerated. Hence, in the design of the compensation filters we do not take the stronger specular echoes into consideration, i.e. throughout this chapter it is assumed that

$$c(k) = w(k) \quad (3.2)$$

where $w(k)$ is a Gaussian process with variance σ_c^2 (cf. Eqn. (1.1)). In order to exploit short-time stationarity and Wiener-filtering, an interval I_j centered around $k=k_j$ from the received signal is gated out yielding

$$z_{k_j}(k) = z(k) p_R(k-k_j) \quad (3.3a)$$

or

$$z_{k_j}(k) = \left\{ \sum_i w(i) s_i(k-i) + n(k) \right\} p_R(k-k_j) \quad (3.3b)$$

where $p_R(k)$ denotes a rectangular window

$$p_R(k) = \begin{cases} 1 & -K/2 \leq k \leq K/2 \\ 0 & \text{else} \end{cases} \quad (3.4)$$

Since the shape of $s_i(k)$ varies slowly, all echoes contributing to the received signal within the interval will have approximately the same shape provided that the length of the interval is not too large i.e.

$$s_i(k) \approx s_{k_j}(k) \quad (3.5)$$

Inserting Eqn. (3.5) in Eqn. (3.3b) yields

$$z_{k_j}(k) \approx \left(\sum_i w(i) s_{k_j}(k-i) + n(k) \right) p_R(k-k_j) \triangleq \tilde{z}_{k_j}(k) p_R(k-k_j) \quad (3.6)$$

If σ_c^2 is constant for all echoes contributing to $z_{k_j}(k)$, $\tilde{z}_{k_j}(k)$ is a stationary random process with covariance $\tilde{r}_{k_j}(k)$ and spectral density $\tilde{R}_{k_j}(f)$ given by

$$\tilde{r}_{k_j}(k) = \sigma_c^2 \left(s_0(k) * s_0(-k) * h(k, x_{k_j}) * h(-k, x_{k_j}) \right) + \sigma_n^2 \delta(k) \quad (3.7)$$

$$\tilde{R}_{k_j}(f) = \sigma_c^2 |S_0(f) H(f, x_{k_j})|^2 + \sigma_n^2 \quad (3.8)$$

where $*$ denotes discrete convolution. The transfer function $H(f, x_{k_j})$ describes the two-way transmission properties of the tissue from the surface to the depth x_{k_j} and is assumed to obey (cf. Eqn. (1.6b))

$$|H(f, x_{k_j})| = e^{-a+\gamma|f|} e^{-\frac{R}{v} x_{k_j}} = e^{-a+\gamma|f|} e^{-\frac{R}{v} k_j} \quad (3.9)$$

where v as usual is the sound velocity and a and γ are the attenuation parameters of the tissue. Figure 3.1 shows a block diagram of the system which produces the gated output $z_{k_j}(k)$. Since this paper concentrates on the impact of the frequency dependent part of the attenuation, the constant a is set to zero in the sequel.

The signal-to-noise ratio between the scattering echoes at a certain depth and the noise is defined by

$$\text{SNR}(k_j) = 20 \log \frac{\tilde{r}_{k_j}(0) - \sigma_n^2}{\sigma_n^2} \quad (3.10)$$

The SNR decreases with depth due to the attenuation. At the surface where the attenuation is zero ($H(f,0)=1$) we obtain

$$\text{SNR}_{\text{surface}} = \text{SNR}(0) = 20 \log \frac{\sigma_c^2 \sum s_0^2(k)}{\sigma_n^2} \quad (3.11)$$

In order to maintain the resolution we must compensate for the loss of higher frequencies caused by $h(k, x_{k_j})$, or equivalently, to regain the signal

$$u_d(k) = \sum_i c(i) s_0(k-i) \quad (3.12)$$

from the observed signal $z(k)$. In an ideal case without measurement noise such a compensation can be done by inverse filtering of the signal $z_{k_j}(k)$, thus using a filter with the frequency function

$$G_{\text{inv}}(f, x_{k_j}) = \begin{cases} \frac{1}{H(f, x_{k_j})} & \text{within passband of transducer} \\ 0 & \text{outside} \end{cases} \quad (3.13)$$

Taking the noise into consideration, the Wiener filter is the optimal linear time-invariant filter (WI-49),

$$G_{\text{wien}}(f, x_{k_j}) = \frac{\tilde{R}_{k_j}(f) - \sigma_n^2}{\tilde{R}_{k_j}(f)} \cdot \frac{1}{H(f, x_{k_j})} \quad (3.14)$$

The Wiener filter requires knowledge about the frequency transfer function $H(f, x)$ for different depths. Often no distinction is made between the frequency function $H(f, x)$ and its amplitude function $A(f, x) = |H(f, x)|$, i.e. the corresponding phase is omitted. The reason for this is that the analysis and methods are based only on the amplitudes of echoes or on the amplitude function of the received ultrasonic signal. The omission of the phase results, of course, in noncausal impulse responses $h(k, x)$, a problem which is often circumvented by a suitable time delay (KA-80, GU-82). Studies have been done to determine the phase analytically (GU-82, KU-81) using a reasonable minimum-phase shift assumption about the tissue giving causal impul-

se responses. A weakness of these studies is that strict conditions are imposed on the amplitude function $A(f, x)$. Since we work in discrete time a numerical method can be used to determine the phase $\Phi(f, x_{k_j})$ from sampled values of the amplitude function. In (OPP-75) such a numerical algorithm is described where a finite, causal and approximately minimum-phase shift sequence $h(k, x_{k_j})$ is obtained from the amplitude function. Once the phase is determined by this procedure the Wiener filter can be calculated from Eqn. (3.14). In Fig. 3.2 causal impulse responses at various depths are shown as obtained by means of this algorithm. These impulse responses are similar to the results reported in (GU-82). Although the tissue is considered to be minimum-phase the Wiener filter is not in general realizable. However, by allowing a suitable time delay arbitrarily good approximations can be obtained.

Each Wiener filter is matched to the spectral content in the corresponding interval I_j . By using a set of filters which are activated sequentially at appropriate points in time, see Fig. 3.3, a time varying frequency dependent filter is obtained. The total filtered output $u(k)$ is given by

$$u(k) = \sum_j \sum_l z_{k_j}(l) g(k-l, x_{k_j}) \quad (3.15)$$

This sum of convolutions are efficiently computed by means of FFT and overlap-add technique.

It is obvious from Eqn. (3.9) that a change in the attenuation coefficient γ can be replaced by a change in the distance x . Using this property, the sequence of compensation filters can be calculated in advance matching a tissue with constant predetermined attenuation coefficient $\gamma = \gamma_0$. Although the filters in the sequence are calculated for equi-distant depths, these can still be used in a situation where the attenuation differs from γ_0 and even varies within the tissue. This situation is easily met by letting the lengths of the switching intervals I_j vary in Eqn. (3.15). If the attenuation is higher than γ_0 in some region, the intervals I_j are chosen shorter and vice versa.

How shall the switching points or, equivalently, the lengths of the intervals I_j be determined? On most ultrasonic equipment the operator determines the amount of TGC, i.e. the frequency independent compensation, manually. Often the TGC is constant over the whole image but on some equipment the image can be divided into regions with different amounts of TGC.

The same procedure can of course be used also for the frequency dependent part of the compensation. The operator controls both the amount of TGC and frequency dependent compensation in order to obtain good resolution together with appropriate brightness in the image. An alternative way is to estimate the attenuation by means of e.g. the methods previously presented. By means of these estimates the amount of TGC and frequency dependent compensation can be automatically determined. Manual compensation has the advantage of being easily controllable but it is difficult to use when the attenuation varies substantially in the tissue. In such a situation automatic compensation is more appropriate.

3. REALIZATION OF THE SEQUENCE FILTERS

A single receiver filter in the sequence $g(k, x_{k_j})$ is realized using a bank of bandpass filters, see Fig. 3.4, a structure which is well-known from short time spectrum analysis (AL-77, ALR-77). The filter-bank structure has several advantages in our specific application. The filter coefficients control the receiver filter directly in the frequency domain. By designing the subfilters in a special way, the over-all filter becomes an interpolating filter in the frequency domain where a small number of parameters control the frequency function independently of each other. Further, the time varying condition is easily met by letting the filter coefficients vary with time, or equivalently, with depth while each filter is held fixed. The filter-bank consists of N bandpass filters each realized by FIR-filters of length L with $L \geq N$. The pulse response of the i :th subfilter is given by

$$g_i(k, x_{k_j}) = \frac{1}{N} f(k) e^{-j2\pi i k/N} \quad 0 \leq i \leq N-1 \quad (3.16)$$

or, in the frequency domain

$$G_i(f, x_{k_j}) = \frac{1}{N} F\left(f - \frac{i}{N}\right) \quad (3.17)$$

i.e. each subfilter is obtained by transposing a lowpass-filter $f(k)$ in frequency. The lowpass-filter $f(k)$ is in the sequel given by

$$\frac{1}{N} f(k) = \frac{1}{N} \left(\frac{\sin \frac{\pi k}{N}}{\frac{\pi k}{N}} \right)^2 p_w(k) \quad 0 \leq k \leq L-1 \quad (3.18)$$

which is a "windowed" FIR approximation of a triangular filter in the frequency domain ($p_w(k)$ denotes the window function) given by

$$\frac{1}{N} F(f) \approx \begin{cases} 1 - N|f| & |f| \leq \frac{1}{N} \\ 0 & \text{else} \end{cases} \quad (3.19)$$

The total pulse response of the filter bank is then formed by a weighted sum of the outputs from the subfilters i.e.

$$g(k, x_{k_j}) = \sum_{i=0}^{N-1} \frac{1}{N} f(i) c_{i_j} e^{-j2\pi i k / N} \quad (3.20)$$

which can be rewritten as

$$g(k, x_{k_j}) = f(k) c_{j_j}(k) \quad (3.21)$$

where $\{c_{j_j}(k)\}_{k=0}^{N-1}$ is the inverse DFT (Discrete Fourier Transform) of the coefficients $\{c_{i_j}\}_{i=0}^{N-1}$. Since the filter $g(k, x_{k_j})$ is assumed to be real-valued the filter weights must be pairwise complex-conjugated fulfilling $c_{N-i_j} = \bar{c}_{i_j}$ (c_{0_j} real-valued). The over-all frequency function of the filter-bank is given by

$$G(f, x_{k_j}) = \sum_{i=0}^{N-1} c_{i_j} G_i(f, x_{k_j}) = \frac{1}{N} \sum_{i=0}^{N-1} c_{i_j} F\left(f - \frac{i}{N}\right) \quad (3.22)$$

By substituting Eqn. (3.19) in Eqn. (3.22), $G(f, x_{k_j})$ can in the interval $\frac{i}{N} \leq f \leq \frac{i+1}{N}$ be expressed as

$$G(f, x_{k_j}) = c_{i_j} \left[1 - N \left(f - \frac{i}{N} \right) \right] + c_{i+1_j} \left[N \left(f - \frac{i}{N} \right) \right] \quad (3.23)$$

The real and imaginary parts of $G(f, x_{k_j})$ are given by

$$\begin{aligned} G(f, x_{k_j}) = & A_{i_j} \left[1 - N \left(f - \frac{i}{N} \right) \right] + A_{i+1_j} \left[N \left(f - \frac{i}{N} \right) \right] + \\ & + j \left(B_{i_j} \left[1 - N \left(f - \frac{i}{N} \right) \right] + B_{i+1_j} \left[N \left(f - \frac{i}{N} \right) \right] \right) \end{aligned} \quad (3.24)$$

with $c_{i_j} = A_{i_j} + jB_{i_j}$. From Eqn. (3.24) it is obvious that both the real and imaginary part of $G(f, x_{k_j})$ will be linearly interpolated between the values A_{i_j} to A_{i+1_j} and B_{i_j} to B_{i+1_j} respectively.

It remains to determine the coefficients c_{i_j} (or A_{i_j} and B_{i_j}). A straightforward method is to calculate the frequency function for the Wiener filter (with a suitable time-delay) at the frequencies $f_i = \frac{i}{N}$, $i = 0, \dots, N-1$ and putting

$$C_{ij} = G_{\text{wien}}(f_i, x_{k_j}) \quad (3.25)$$

The total frequency function is then a **linearly interpolated approximation** of the Wiener filter. Another approach to the problem is to consider the coefficients C_{ij} as free parameters and to choose these in such a way that the error function

$$E[\epsilon(k)^2] \triangleq E\left[\left(u_d(k) - u_{k_j}(k)\right)^2\right] = E\left[\left(u_d(k) - \sum_i C_{ij} \varphi_{ij}(k)\right)^2\right] \quad (3.26)$$

is minimized, where $\varphi_{ij}(k)$ denotes the output from $g_i(k, x_{k_j})$. It is easily verified (ER-80) that the **least squares solution** is given by the matrix relation

$$\underline{C}_j = \underline{R}^{-1} \underline{b} \quad (3.27)$$

where

$$\underline{C}_j = [C_{0j}, \dots, C_{N-1j}]^T \quad (3.28)$$

$$\underline{R} = E[\underline{\varphi}_j(k) \underline{\varphi}_j^T(k)] \quad (3.29)$$

$$\underline{b} = E[u_d(k) \underline{\varphi}_j(k)]^T \quad (3.30)$$

$$\underline{\varphi}_j(k) = [\varphi_{0j}(k), \dots, \varphi_{N-1j}(k)]^T \quad (3.31)$$

This solution gives the optimum coefficients C_{ij} , but the solution requires more calculations as compared to those in Eqn. (3.25). The computational load can, however, be decreased by observing that only some of the filters in the filter-bank are active while the coefficients for other filters are almost identical to zero. This fact follows from Eqn. (3.14) where it is easily seen that $G_{\text{wien}}(f, x_{k_j}) \approx 0$ at frequencies where the noise energy is dominant over the signal energy, i.e. outside the frequency range of the transducer pulse. It is thus only necessary to determine the coefficients for the subfilters inside the frequency range of the transducer while the remaining ones can be set to zero.

4. EXAMPLES

Filter sequences which approximate the Wiener filter at equidistant depths in a tissue with predetermined fixed attenuation have been calculated using the filter structure described above. The amplitude functions are shown in Figs. 3.6 and 3.7 for these depths. The filters have been calculated with the following presuppositions.

Transducer: A broadband transducer pulse was used with an almost constant amplitude function in the frequency interval [1.7, 4.8] MHz, see Fig. 3.5.

Filterbank subfilters: The filter sequences were formed by means of 7 approximately triangular subfilters covering the frequency interval [1.6, 4.9] MHz. This corresponds to $N=37$ where only filters with indices $i=(3-9)$ and $(28-34)$ are active. Each subfilter has a length of 87 samples. Observe that the transducer frequency range is covered with this choice of filters.

Window function: A standard Hamming window of length $L=87$ was used in the approximation of the triangular subfilters.

Tissue: The tissue was chosen to have an attenuation coefficient $\gamma=0.1$ [Neper/cm MHz] with $\beta=1$. This corresponds to real values in many tissues.

SNR: The $\text{SNR}_{\text{surface}}$ has been varied from 40dB to 80dB in Figs. 3.6a,b,c and 3.7a,b,c (measured data is often found in this interval) while Figs. 3.6d and 3.7d show the amplitude functions in absence of measurement noise.

Relative distance: The relative distance between two adjacent filters in the sequence was chosen to 0.25 cm, but only every tenth is plotted which gives the relative distance $(x_{k_{j+10}} - x_{k_j}) = 2.5$ cm for the plotted amplitude functions.

Linear interpolating approximations of the Wiener filter are shown in Fig. 3.6 while the least-squares approximations are shown in Fig. 3.7. Note the upper limit of the amplitude functions that occur in Figs. 3.6a-c and 3.7a-c but not in 3.6d and 3.7d. This limitation is due to the signal-to-noise ratio which is defined by $\text{SNR}_{\text{surface}}$. The filters follow the function $e^{\gamma|f|x}$ as long as the signal is stronger than the noise, e.g. up to the frequency where the amplitude function reaches a value that approximately is equal to $\text{SNR}_{\text{surface}}$. Above this frequency, the noise is increasingly dominating due to the progressive attenuation, thus causing the Wiener filter to decrease with frequency. Only small differences can be noted in the amplitude functions between the linear interpolating and least-squares approximations; small dips are found in some of the amplitude functions in Fig. 3.7. These dips are explained in Fig. 3.8 in which both the linear inter-

polating and least-squares approximations are plotted together with the true Wiener filter for a single tissue depth. The interpolating filter does not follow the true Wiener filter as well as the least-squares filter with respect to the dip but has, on the other hand, a smoother behaviour. More importantly, no significant difference has been found when applying the filters to simulated ultrasonic signals, see Fig. 3.21.

5. SIMULATION RESULTS

A computer phantom B-scan image "measured" by a 64-line linear array transducer system is generated by means of the model from part 1, see Fig. 3.9. The aim is to illustrate the impact of frequency dependent attenuation compensation on the resolution as well as on the background texture. For this purpose the tissue is simulated to contain thin "blood vessels" at various depths surrounded by homogeneous tissue containing only scattering echoes. Specular reflectors are placed at predetermined positions simulating blood vessels. The scattering variance is considered to be piecewise constant; $\sigma_c = \sigma_0$ inside a blood vessel and $\sigma_c = 2\sigma_0$ outside. The attenuation coefficient γ is chosen to 0.1 [Np/cm MHz]. An ideal image with no attenuation or measurement noise is used as a reference in order to judge the quality of different methods, see Fig. 3.12. In this image no decrease in resolution or distortion of the background texture is obtained. The frequency content of the incident pulse is unaffected. The TGC-compensated B-scan image of the computer phantom without measurement noise is shown in Fig. 3.10. By comparing Fig. 3.10 and Fig. 3.12 it is obvious that the decrease in resolution as well as background texture distortion is considerable at large depths in the tissue due to the attenuation and frequency independent compensation. The gradual decrease in resolution with depth can also be studied in the A-mode registrations along selected lines in the TGC-compensated signals, Fig. 3.11.

Manual Selection of Switching Intervals

Figures 3.10 and 3.12 illustrate the limitations of frequency independent compensation (TGC) in the absence of measurement noise. We now apply the filter sequence in Fig. 3.6d to the received signals. The switching intervals I_j are of equal length, which are manually selected to give good restoration. The optimum length of each interval is, of course, 0.25cm since the filters in Fig. 3.6d were calculated for a tissue with attenuation coefficient $\gamma=0.1$ for equidistant intervals of 0.25cm. This length corresponds to 33 samples at a sampling rate of 20 MHz and a sound velocity of 1540 m/s. Since the length of each received signal along a line in the image contains 2048 samples, the sequence must contain at least 62 filters. The restoration of the image, see Fig. 3.13, is almost perfect. The resolution of the image corresponds to the resolution in the ideal image and the background texture is almost identical to the ideal texture. The resolution can also be studied in detail in the A-mode registrations, see Fig. 3.14

(cf. Fig. 3.11). The almost perfect restoration is due to the absence of measurement noise since no limitations of the inverse filtering are set.

When adding measurement noise to the received signal the filter sequence in Fig. 3.6d is mismatched to the situation, see Fig. 3.15. The image is corrupted when the frequency dependent compensation becomes more accentuated at high frequencies deep in the tissue. In this case, the mismatch is considerable since the filter sequence is designed for a noise-free situation on a signal with SNR equal to only 40 dB at the surface. Due to the strong amplification in the upper part of the transducer's frequency band the noise dominates the image deep in the tissue and the original texture and anatomy are lost.

In Figs. 3.16 and 3.17 images are shown where the filter sequences are matched to the noise level. Figure 3.16 shows the results when the SNR at the surface is 60 dB and thus the filter sequence in Fig. 3.6b is used together with manual determination of switching interval length. The corresponding TGC image is also shown and it is obvious that the resolution and texture is superior when using frequency dependent compensation as compared to normal TGC compensation. The corresponding results when the SNR at the surface is only 40 dB is shown in Fig. 3.17. The filter sequence in Fig. 3.6a is used, but here it is only possible to restore the image partially due to the high noise level. Nevertheless, it is obvious that frequency dependent compensation is superior to TGC also in this situation.

Automatic Selection of Switching Intervals

Finally the frequency dependent compensation is applied to a situation where the attenuation in the simulated tissue varies with depth. Thus the length of the switching intervals I_j must also vary with depth. In this case manual selection of the switching intervals is difficult to perform. Instead, automatic determination of the switching interval lengths based on attenuation estimates can be used which is illustrated below.

A tissue with the attenuation profile given in Fig. 3.18 was simulated. The tissue was chosen to have the attenuation coefficient $\gamma=0.075$ [Np/cm MHz] outside and 0.15 [Np/cm MHz] inside the interval $k=(900,1400)$ along all the 64 lines in the image. By applying the time domain estimation method from Part 2 to the received signals a time-varying estimate of the attenuation is obtained by averaging of the 64 estimates, see Fig. 3.18. From this es-

timate the switching points are easily determined in the following manner. The attenuation between the surface and at k samples deep in the tissue is given by

$$\Gamma(k) = \sum_{\ell=0}^k \gamma(\ell) \quad (3.32)$$

The filter sequences have been calculated in advance for a tissue having $\gamma=0.1$ [Np/cm MHz] with relative distance 0.25 cm corresponding to 33 samples. In order to obtain correct compensation the j :th filter should be connected at the depth k where

$$\sum_{\ell=0}^k \hat{\gamma}(\ell) = (j-1) \sum_{\ell=1}^{33} 0.1 \quad (3.33)$$

where $\hat{\gamma}$ denotes the estimated attenuation coefficient. The resulting compensated images together with TGC images are shown in Fig. 3.19 and Fig. 3.20 for signals having $\text{SNR}_{\text{surface}}$ equal to 60 and 40 dB respectively. Again we observe improved resolution and background texture when using frequency dependent compensation. Comparing these images with the corresponding images in Fig. 3.17b and 3.18b where the attenuation was constant and the switching points were determined manually, the results indicate that varying attenuation can be effectively compensated for by using frequency dependent compensation. The difference between manual compensation (for constant attenuation) and automatic compensation (for varying compensation) is negligible.

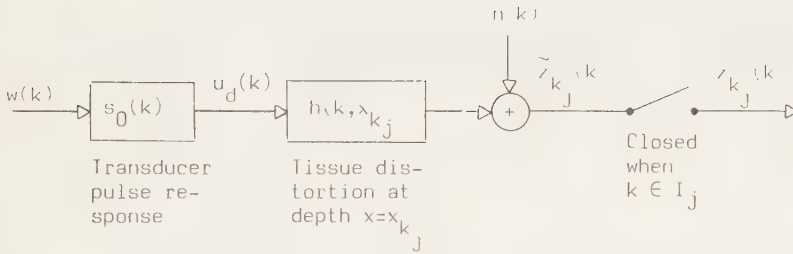


Fig. 3.1 Block diagram of stationary system model forming the gated output $z_{k,j}$.

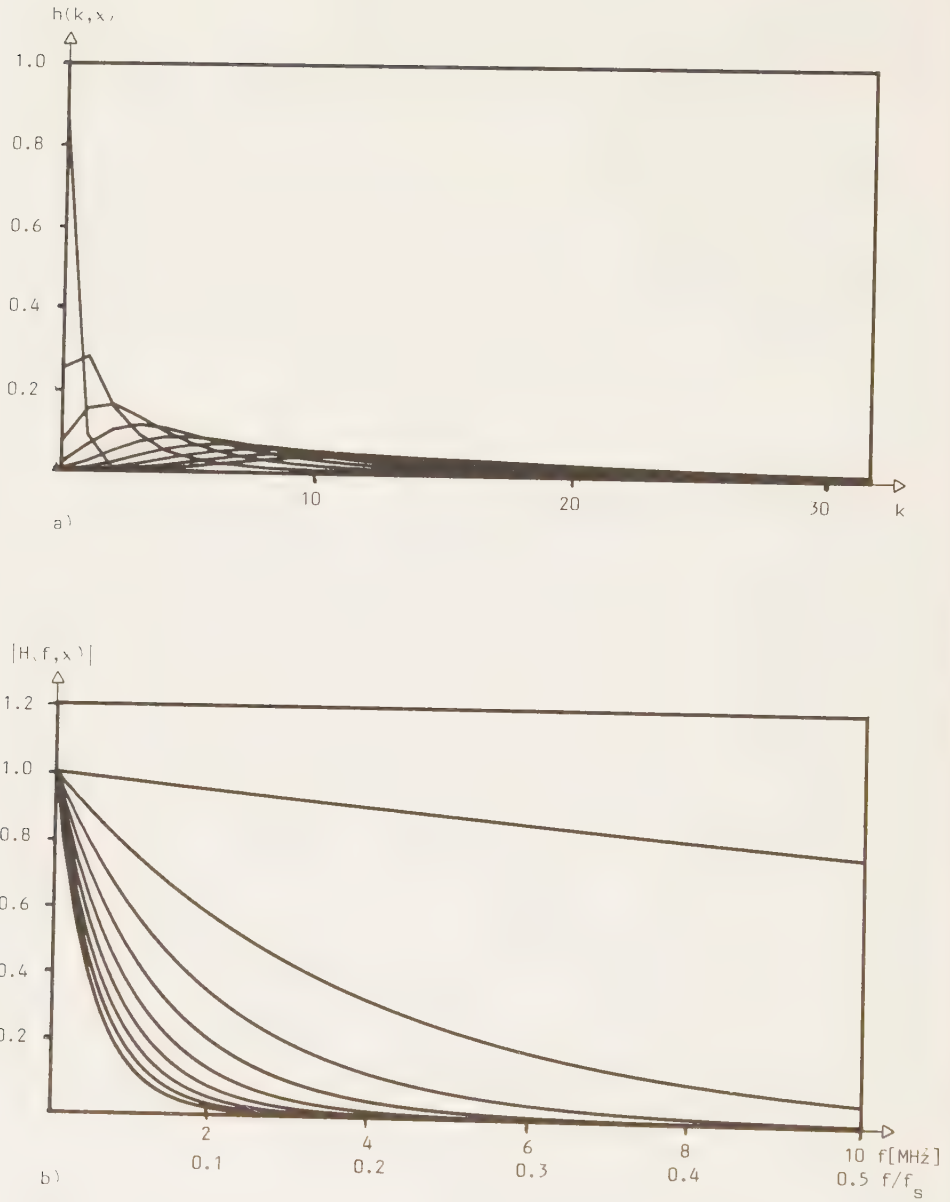


Fig. 3.2 The causal pulse responses $h(k, x)$ (a) with the corresponding frequency functions $|H(f, x)| = e^{-\gamma|f|x}$ with $\gamma \cdot x$ varied from 0.025 to 2.025 Np/MHz in steps of 0.25 Np/MHz (b).

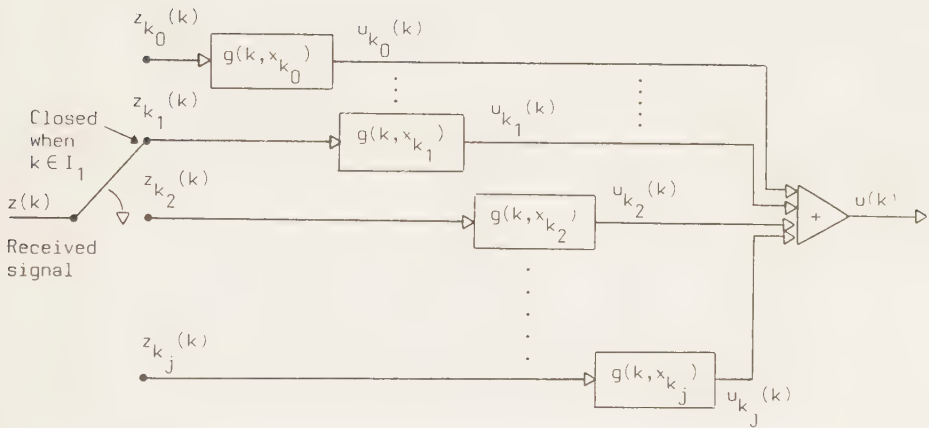


Fig. 3.3 Successive switching of the filters in the receiver filter sequence.

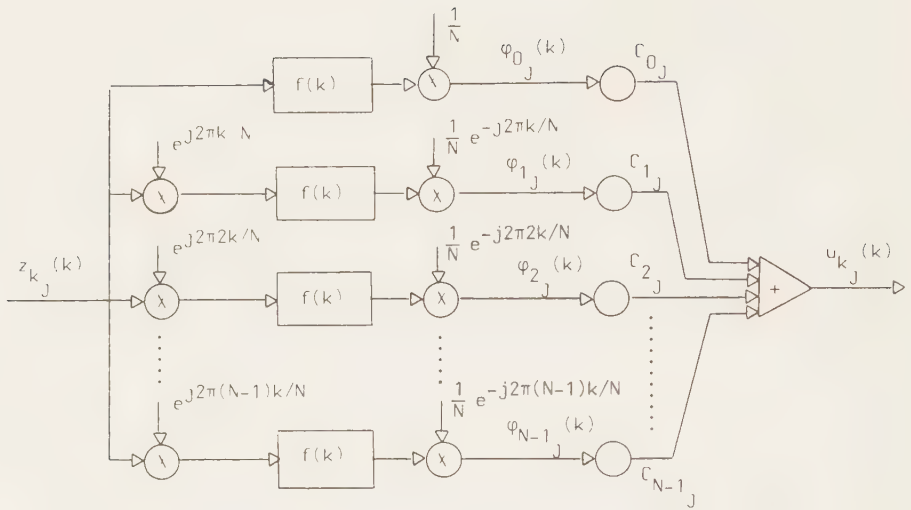
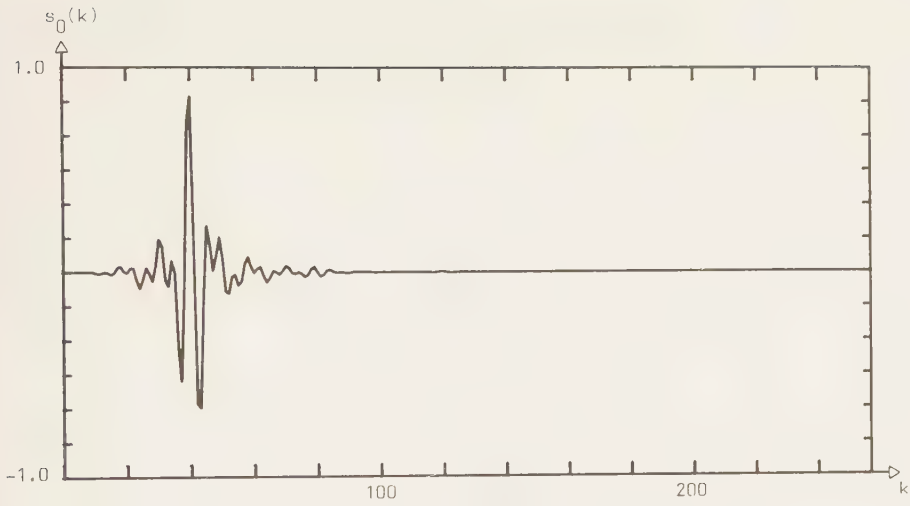
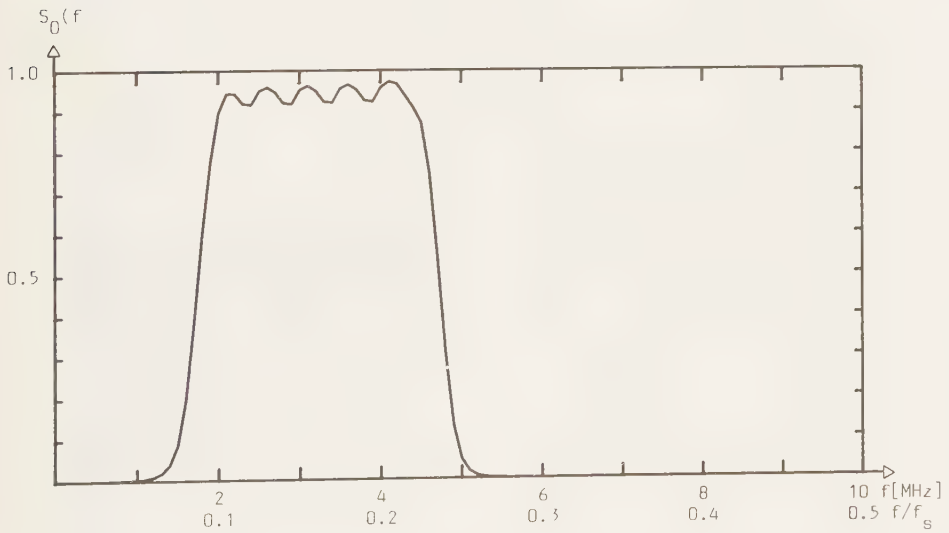


Fig. 3.4 Filter bank realization of a single filter $G(f, x_{k_j})$ in the receiver filter sequence.



a)



b)

Fig. 3.5 The transducer pulse $s_0(k)$ (a) with the corresponding frequency function $S_0(f)$ (b).

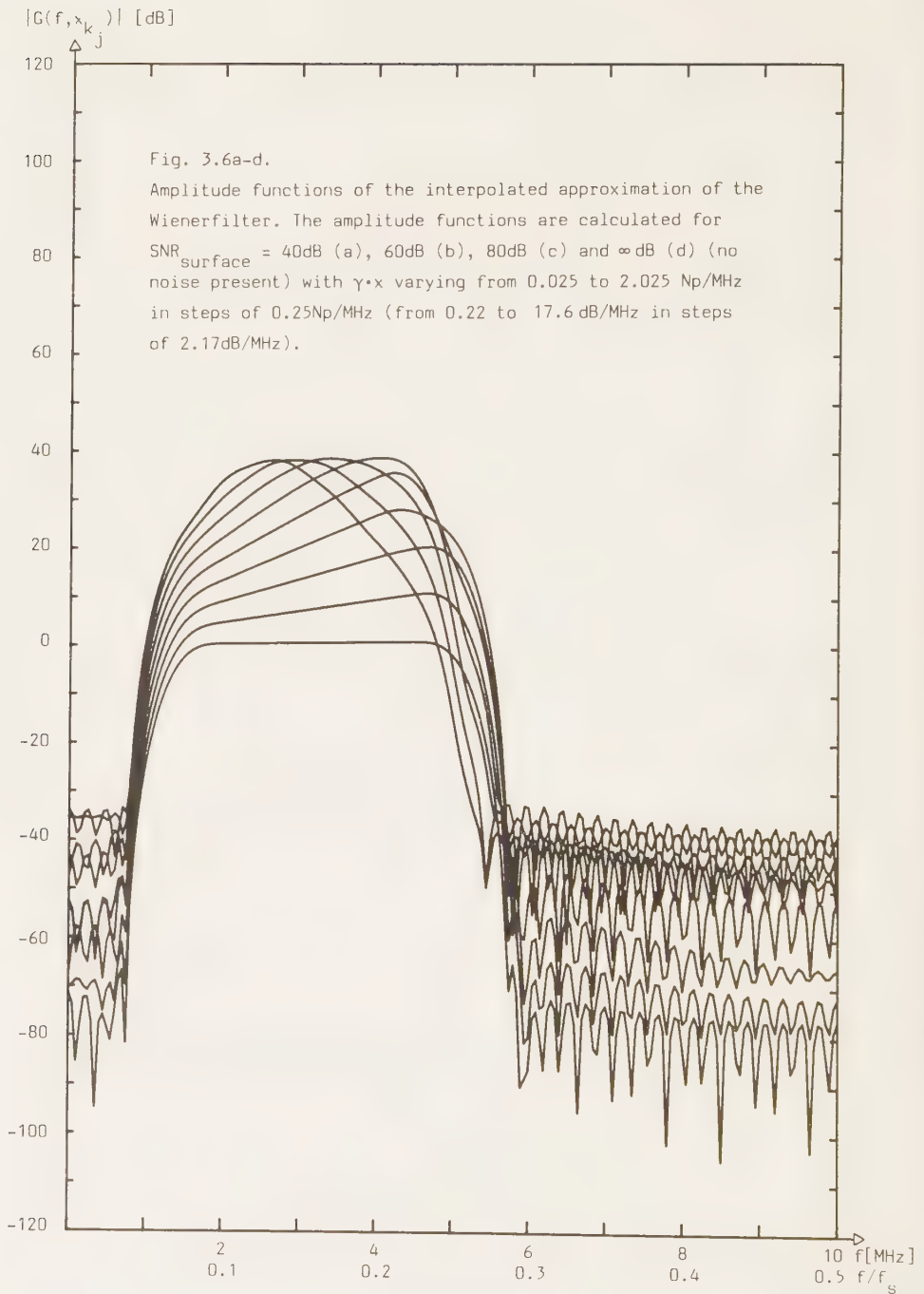


Fig. 3.6a. $\text{SNR}_{\text{surface}} = 40\text{dB}$.

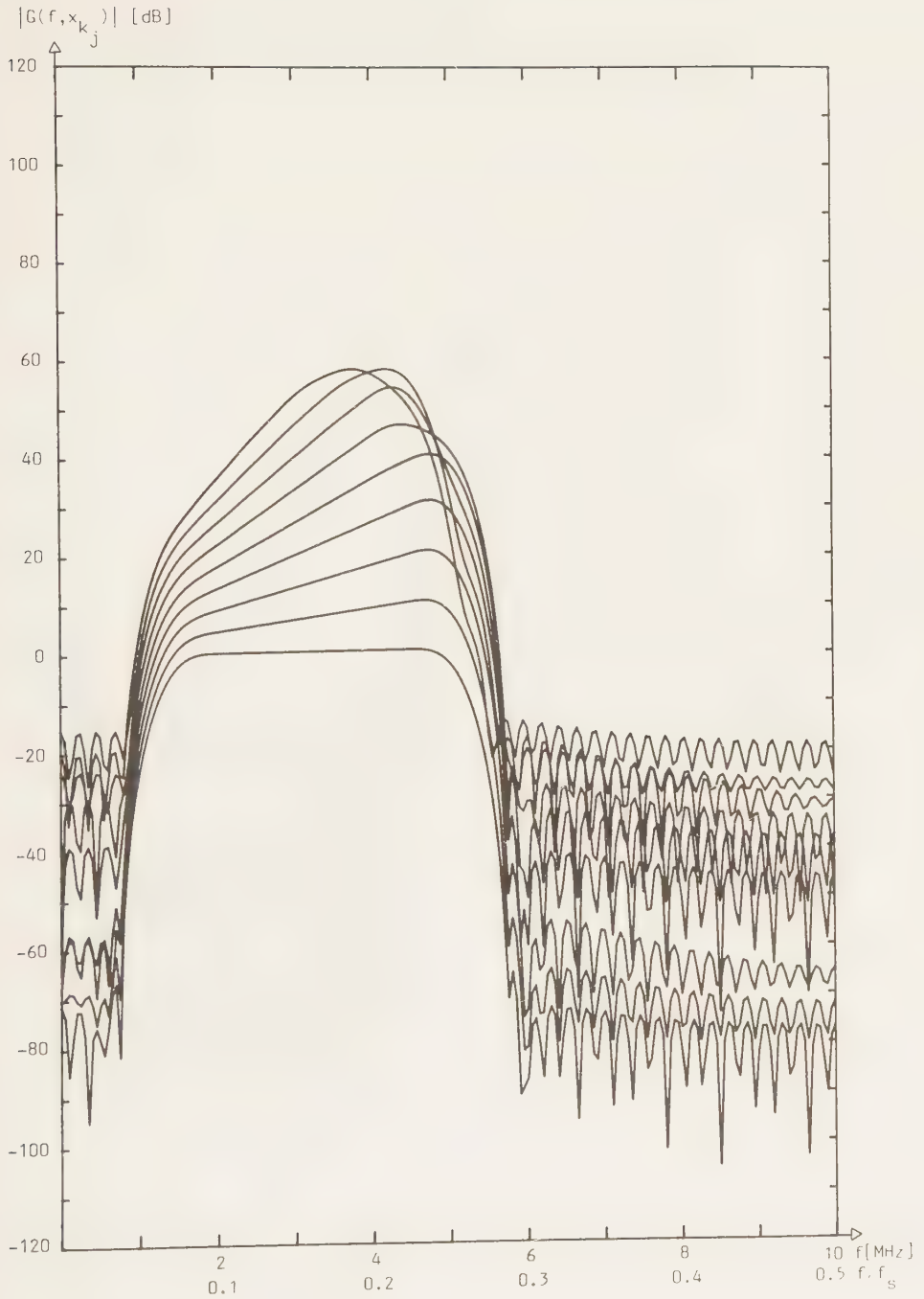


Fig. 3.6b. $\text{SNR}_{\text{surface}} = 60\text{dB}$.

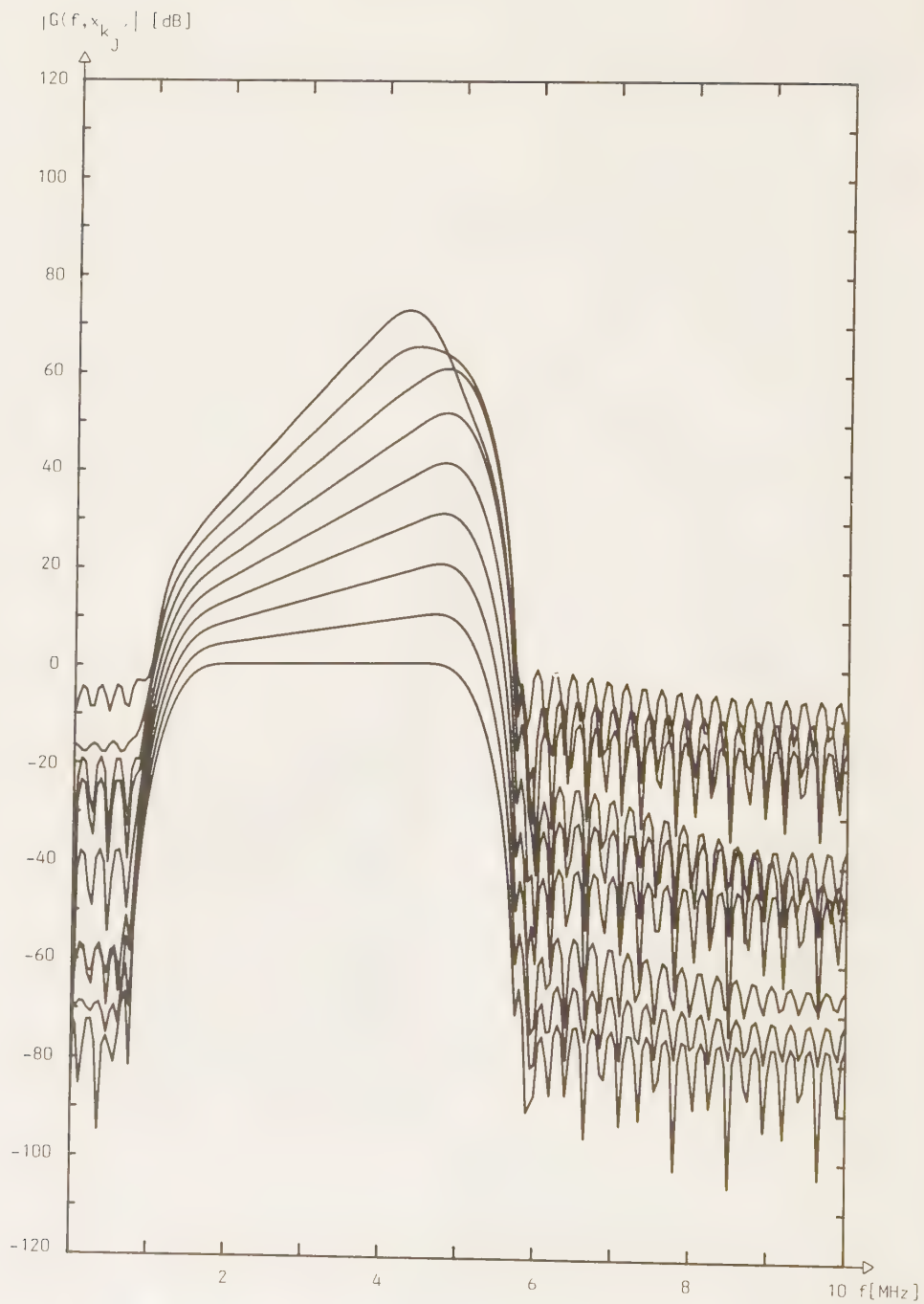


Fig. 5.6c. $\text{SNR}_{\text{surface}} = 80\text{dB}$.

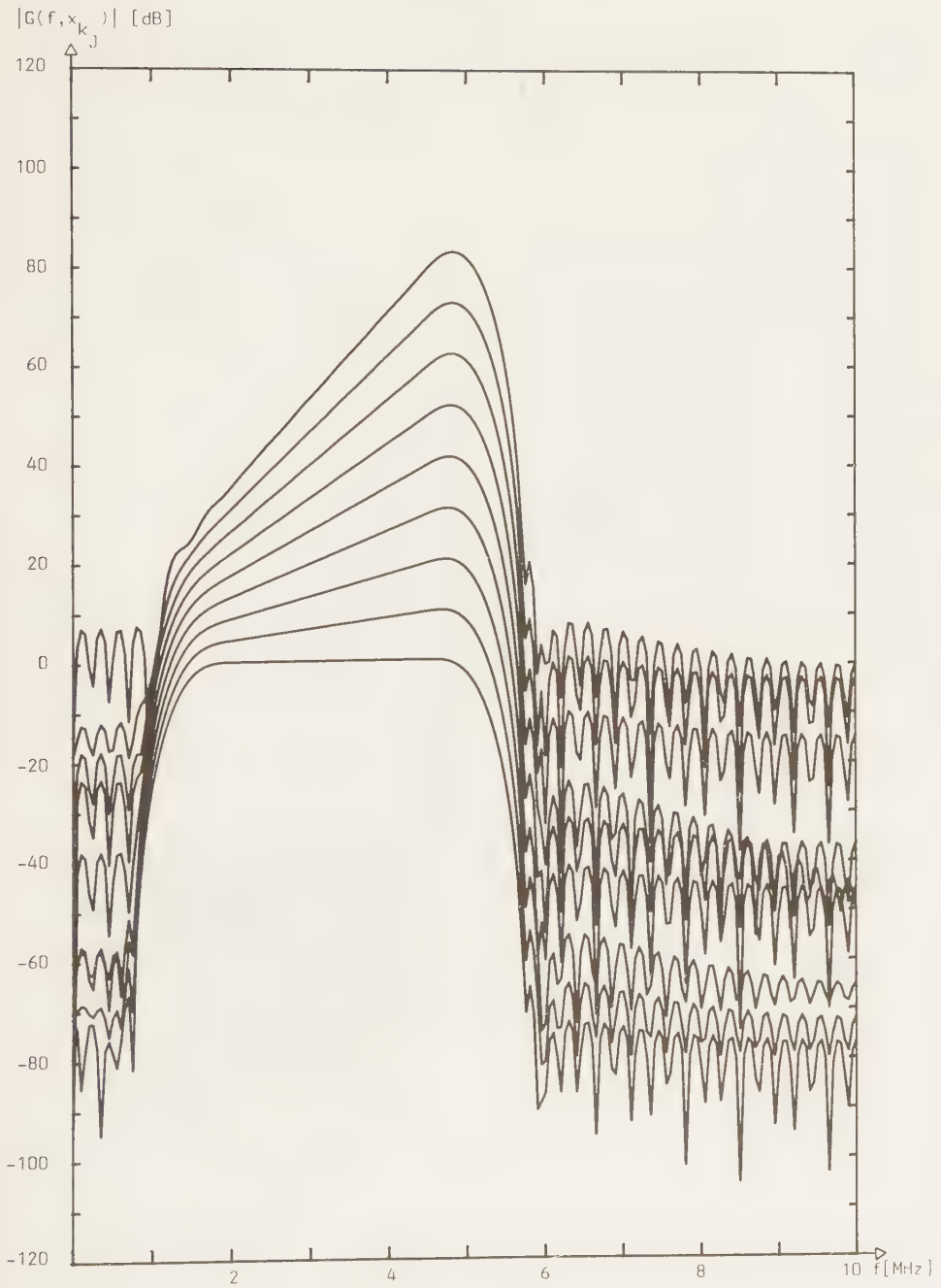


Fig. 3.6d. $\text{SNR}_{\text{surface}} = \infty$ dB (no noise present).

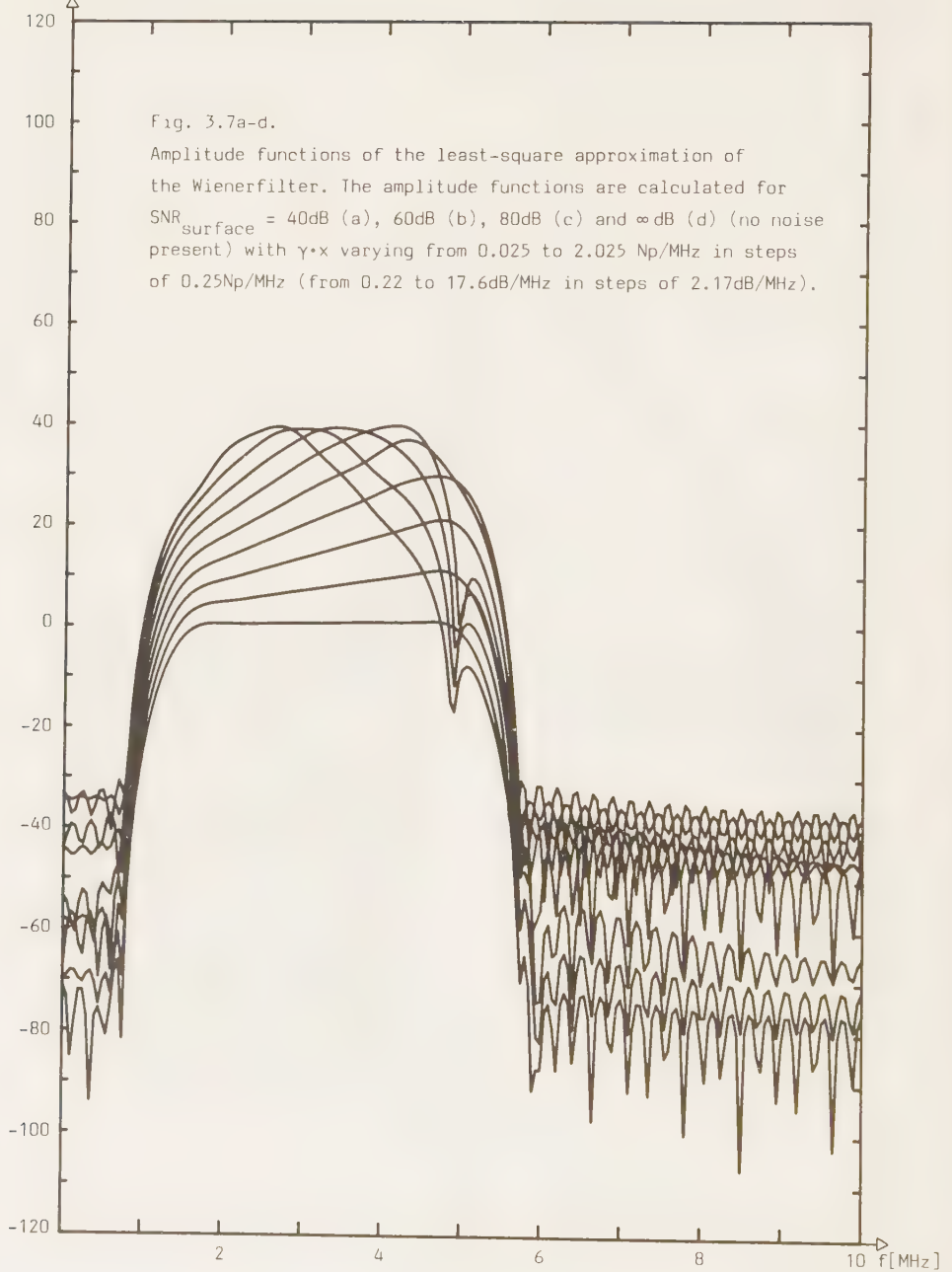
$|G(f, x_k)| \text{ [dB]}$


Fig. 3.7a. $\text{SNR}_{\text{surface}} = 40\text{dB}$.

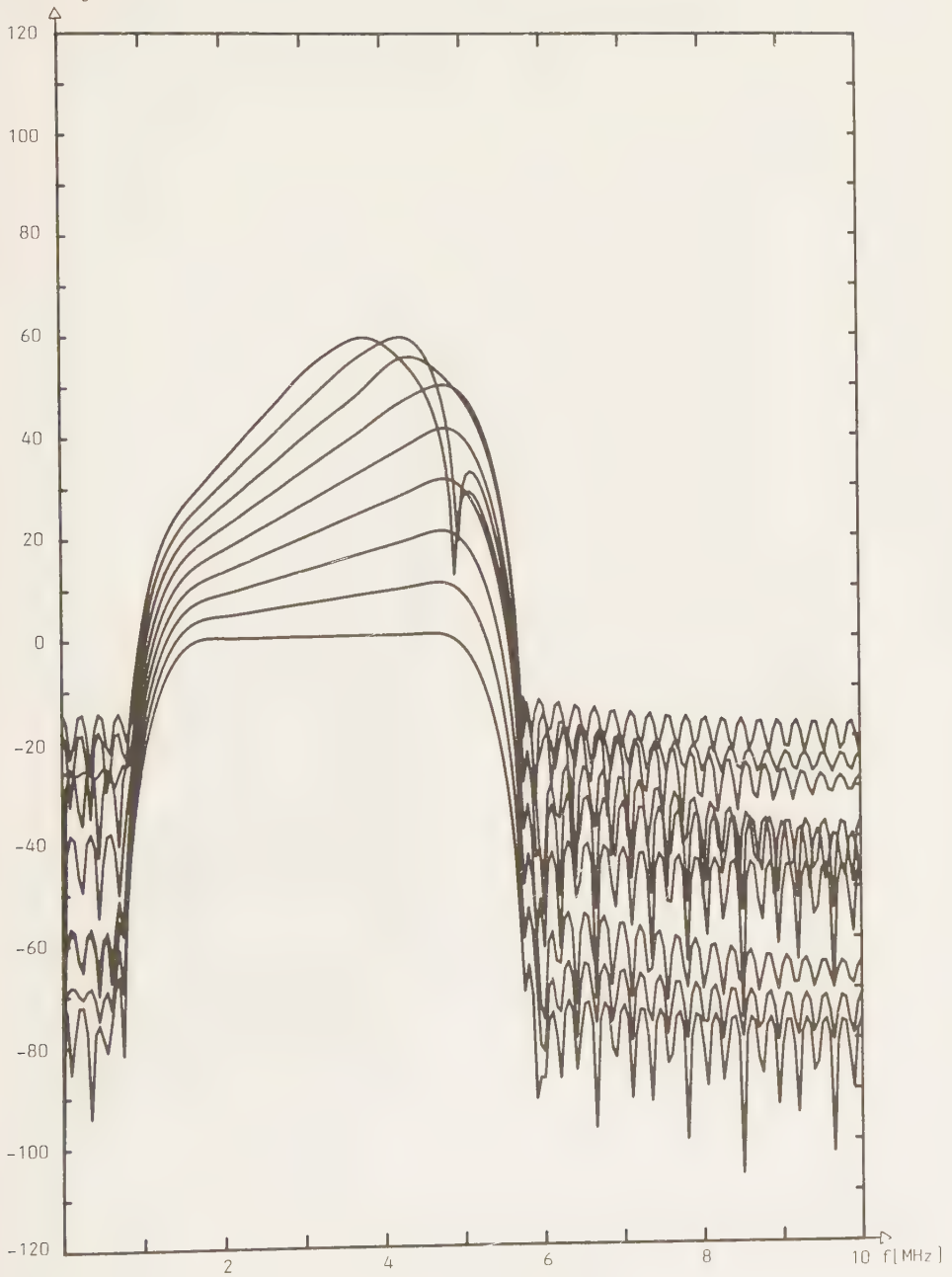
$|G(f, x_{k_j})| \text{ [dB]}$


Fig. 3.7b. $\text{SNR}_{\text{surface}} = 60\text{dB}$.

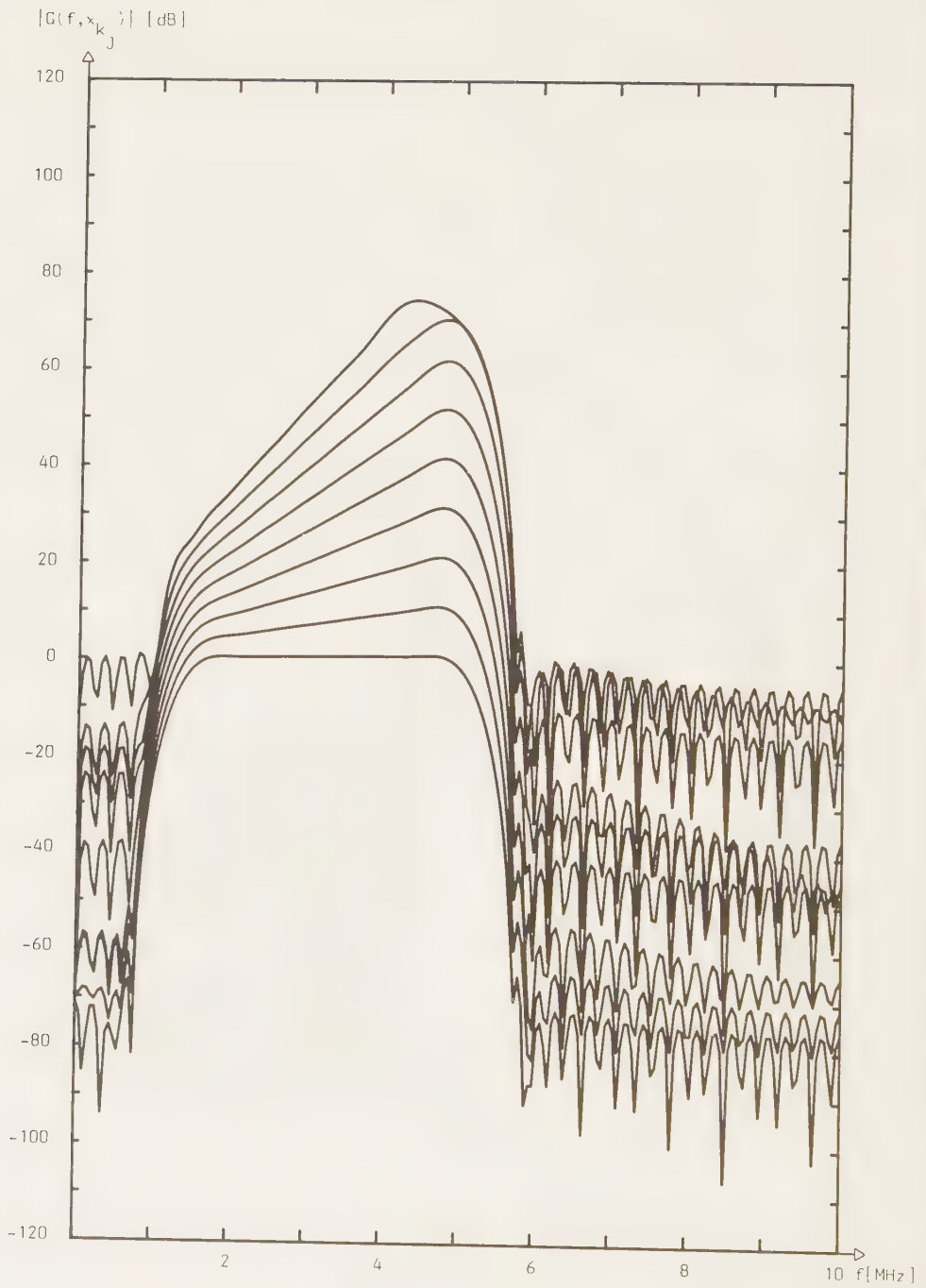


Fig. 3.7c. $\text{SNR}_{\text{surface}} = 80\text{dB}$.

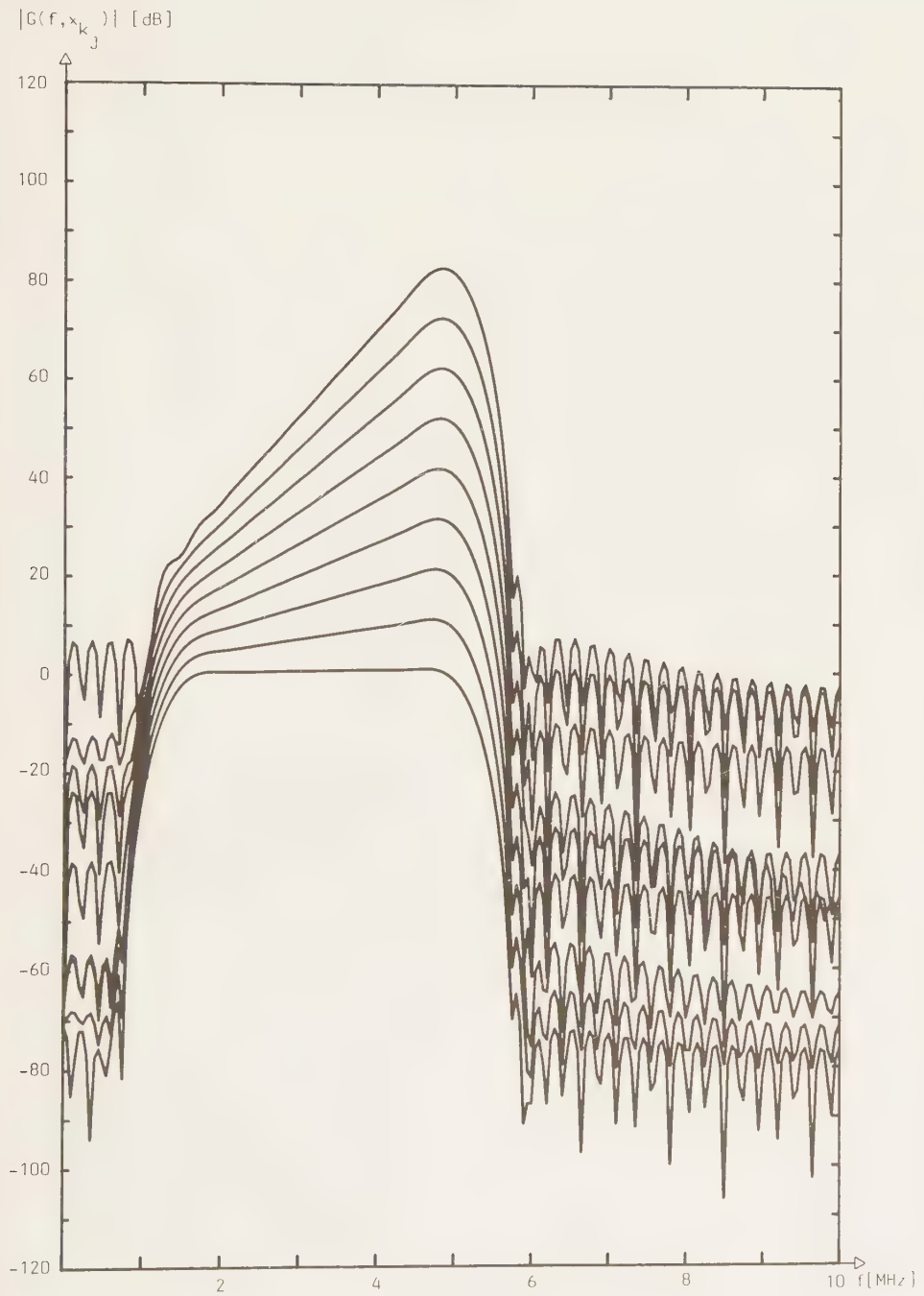


Fig. 3.7d. $\text{SNR}_{\text{surface}} = \infty \text{ dB}$ (no noise present).

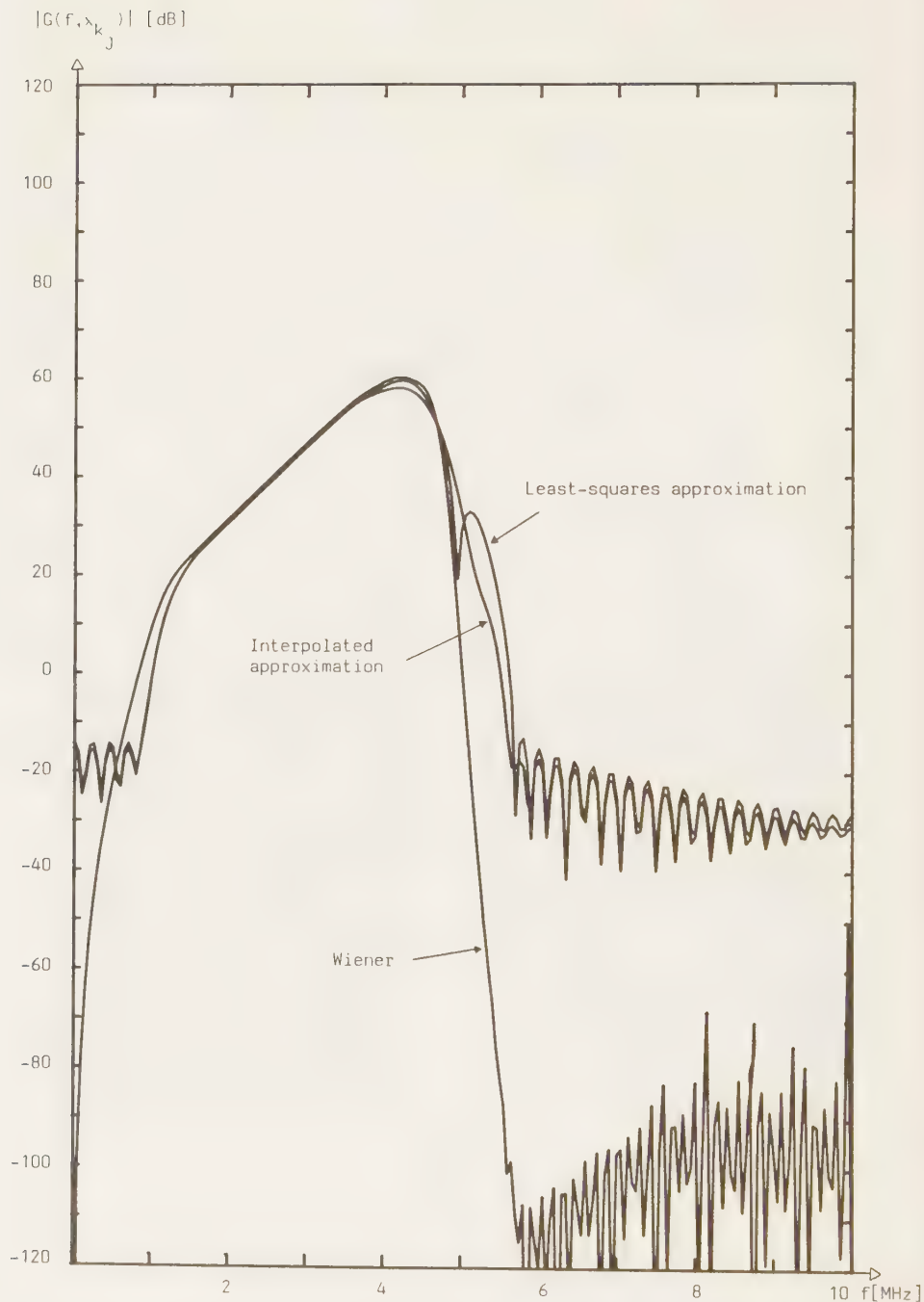


Fig. 3.8 Interpolating approximation, least-squares approximation and true Wienerfilter amplitude functions. $\text{SNR}_{\text{surface}} = 60\text{dB}$ and $\gamma \cdot x = 1.775 \text{ Np/MHz}$ (15.4 dB/MHz).

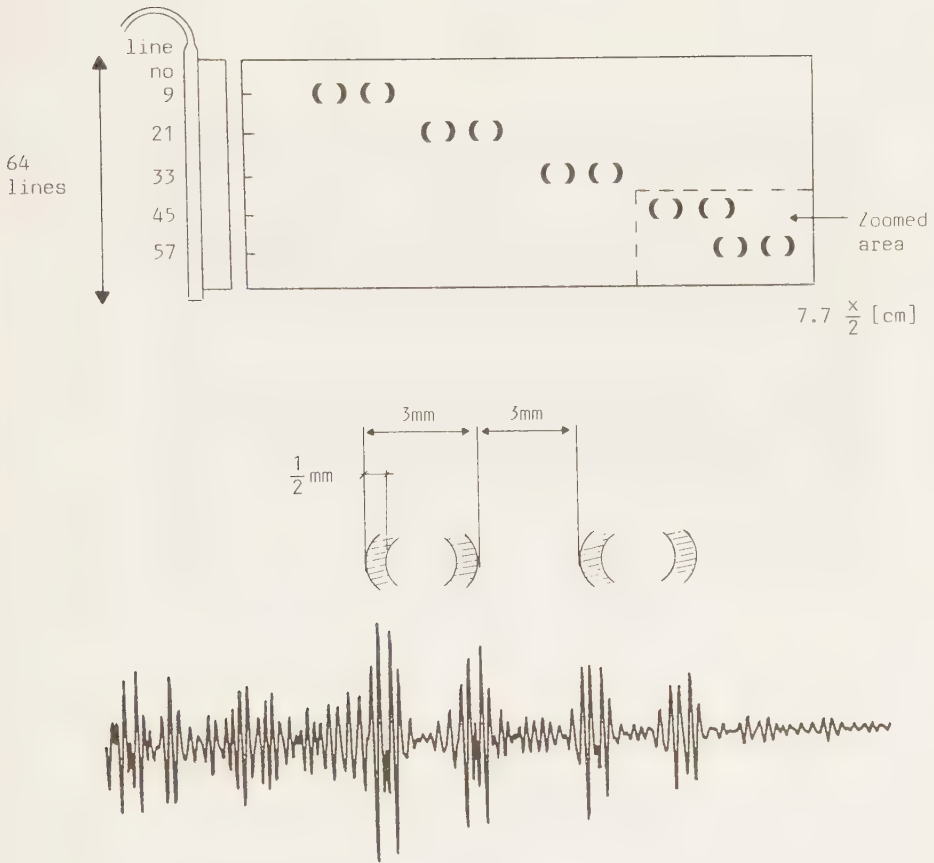


Fig. 3.9 Schematic illustration of the computer phantom with "blood-vessels" at predetermined positions. Zoomed area dotted.

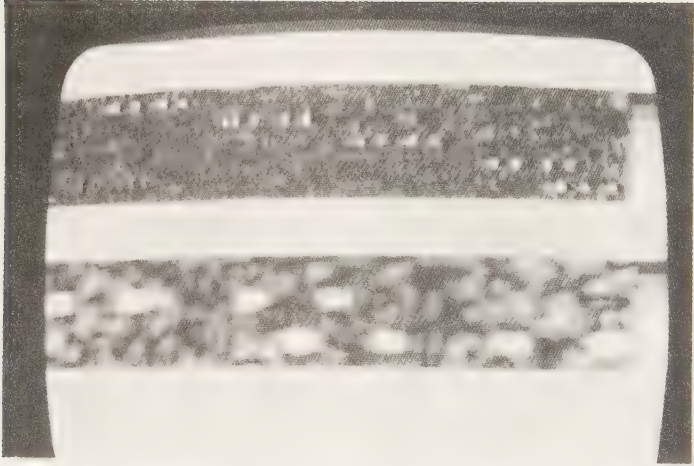


Fig. 3.10 a) TGC B-scan of computer phantom.
b) Zoomed image of dotted area (see Fig. 3.9).

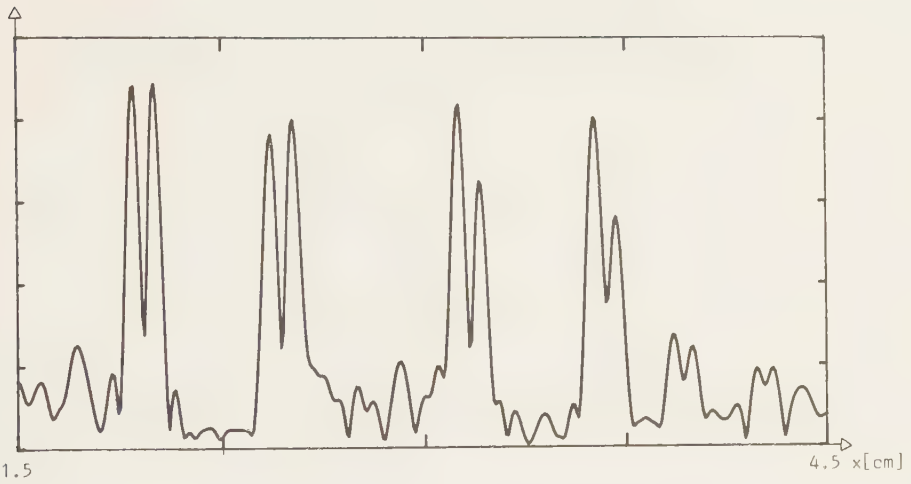


Fig. 3.11a. Line 9.

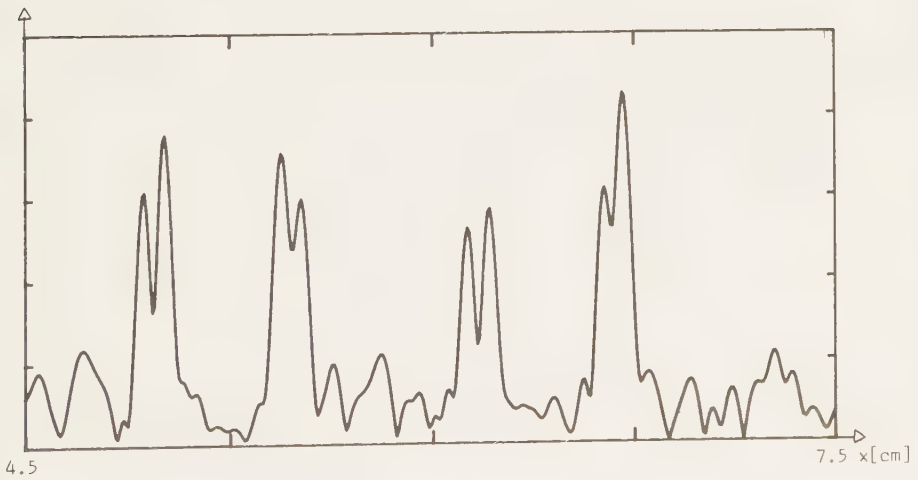


Fig. 3.11b. Line 21.

Fig. 3.11a-e.

TGC A-mode registrations along lines 9 (a), 21 (b), 33 (c), 45 (d) and 57 (e) at the depths where the blood-vessels occur.

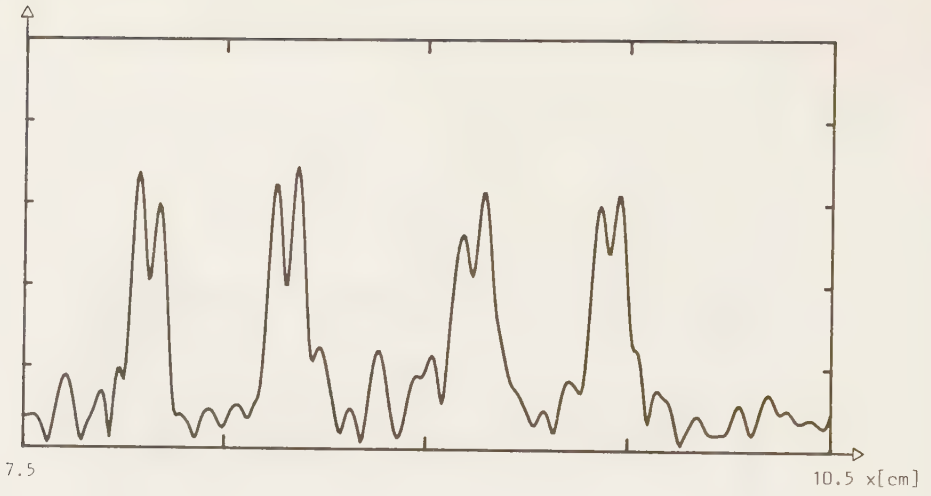


Fig. 3.11c. Line 33.

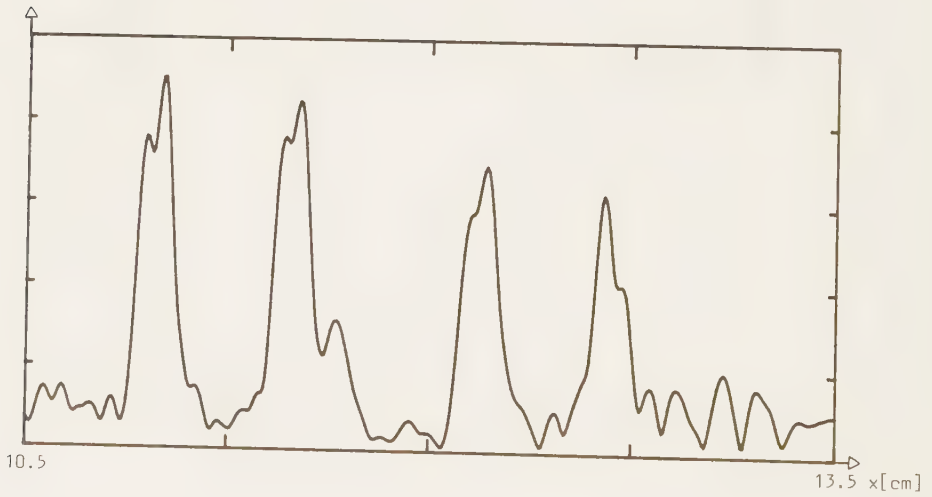


Fig. 3.11d. Line 45.

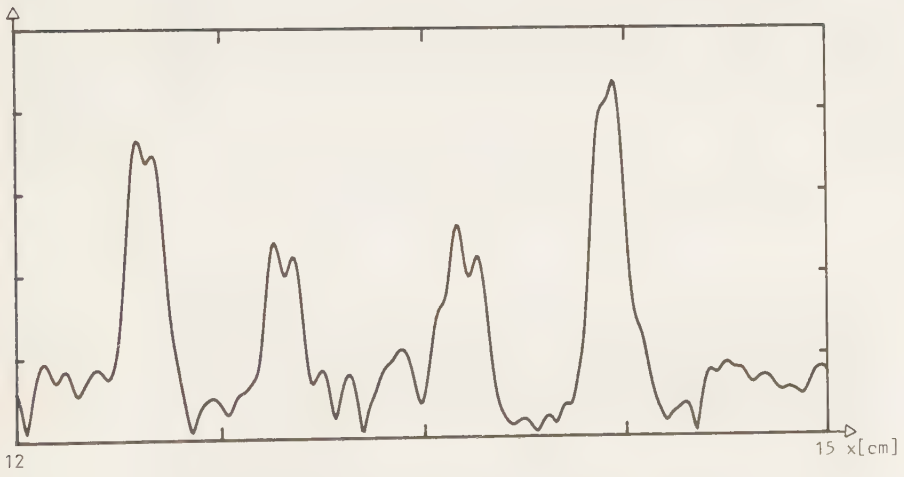


Fig. 3.11e. Line 57.

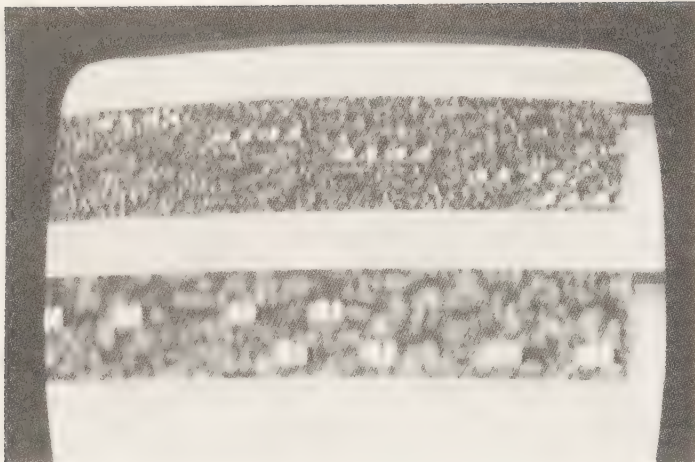


Fig. 3.12 a) Ideal B-scan image (no attenuation in the tissue).
b) Dotted area zoomed.

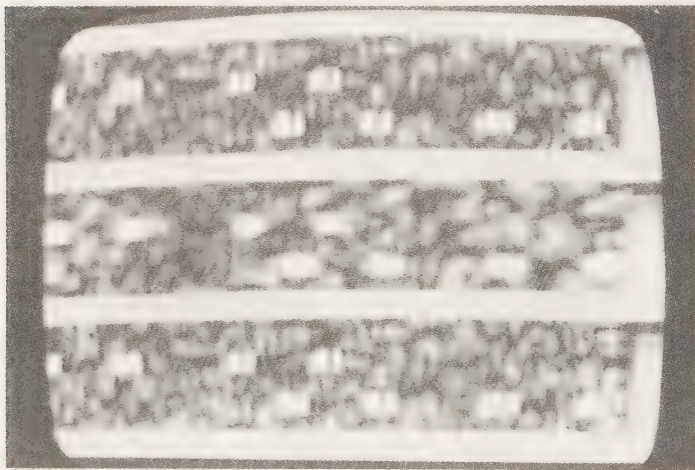


Fig. 3.13 Comparison of the resolution in the zoomed area.
a) Zoomed ideal B-scan image.
b) Zoomed TGC B-scan image.
c) Zoomed frequency dependent compensated B-scan image.
(Frequency compensation using the filters in Fig. 3.7d.)

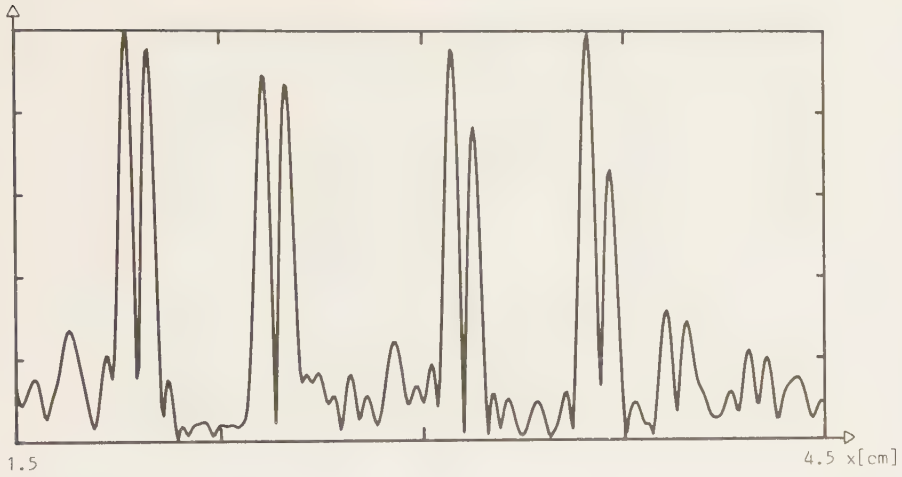


Fig. 3.14a. Line 9.

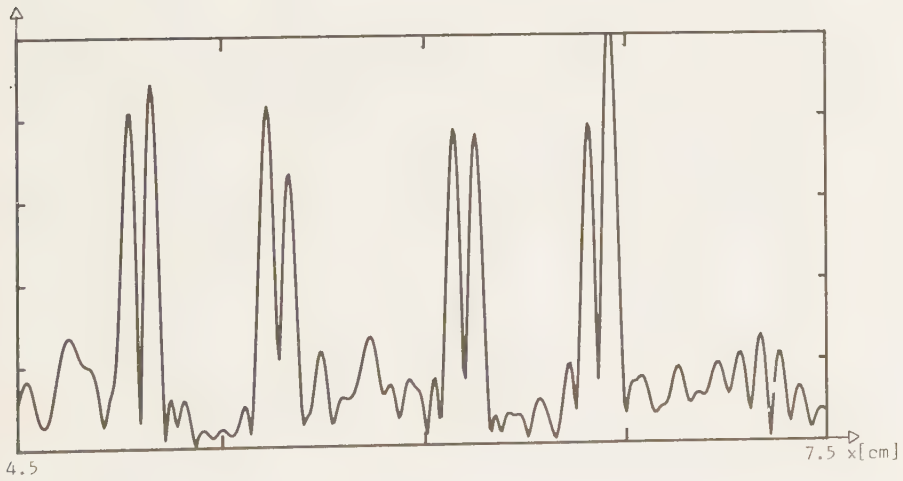


Fig. 3.14b. Line 21.

Fig. 3.14a-e.

Frequency dependent compensated A-mode registrations along lines 9 (a), 21 (b), 33 (c), 45 (d) and 57 (e) at the depths where the blood-vessels occur.

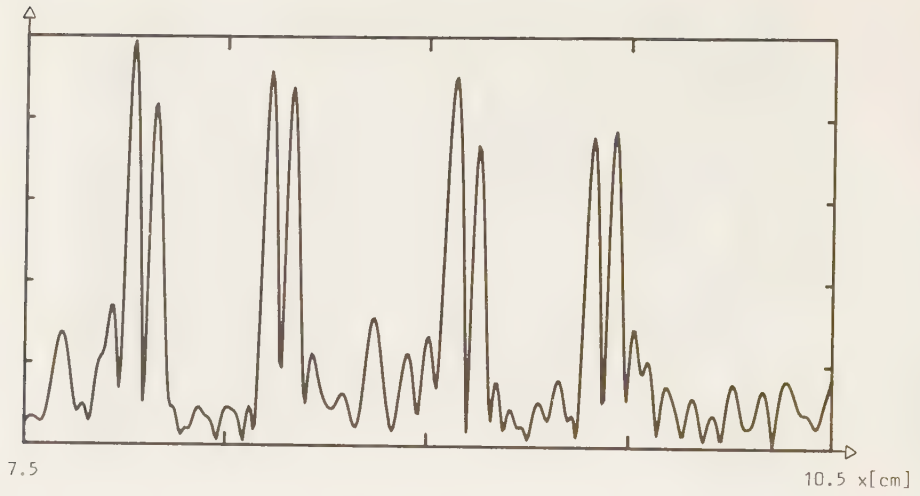


Fig. 3.14c. Line 33.

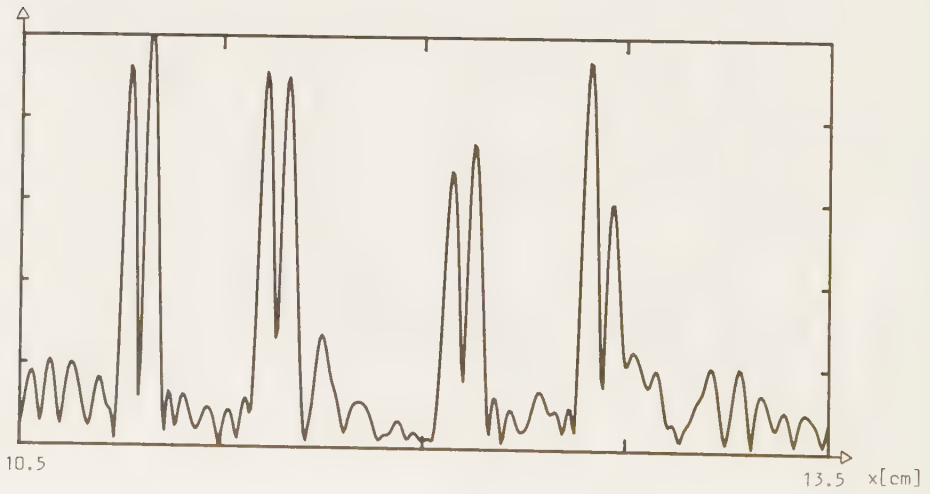


Fig. 3.14d. Line 45.

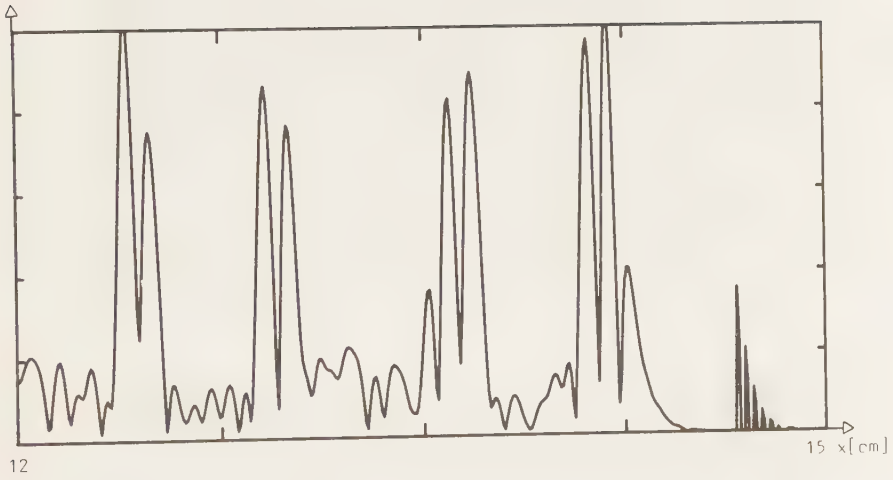


Fig. 3.14e. Line 57.

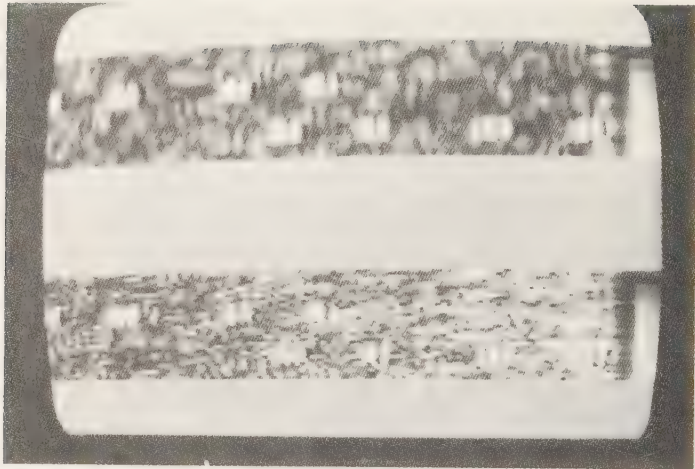


Fig. 3.15 Illustration of the impact of noise. Frequency dependent compensation using the filter in Fig. 3.7d is performed on a signal without noise (a) and a signal with $\text{SNR}_{\text{surface}} = 40\text{dB}$ (b).

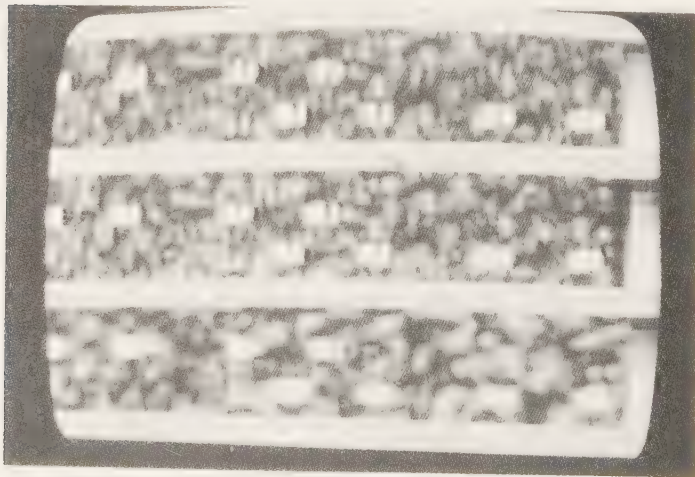


Fig. 3.16 a) Zoomed ideal B-scan image.
 b) Frequency dependent compensated B-scan image with $\text{SNR}_{\text{surface}} = 60\text{dB}$ (the filter in Fig. 3.6b is used).
 c) TGC B-scan image $\text{SNR}_{\text{surface}} = 60\text{dB}$.

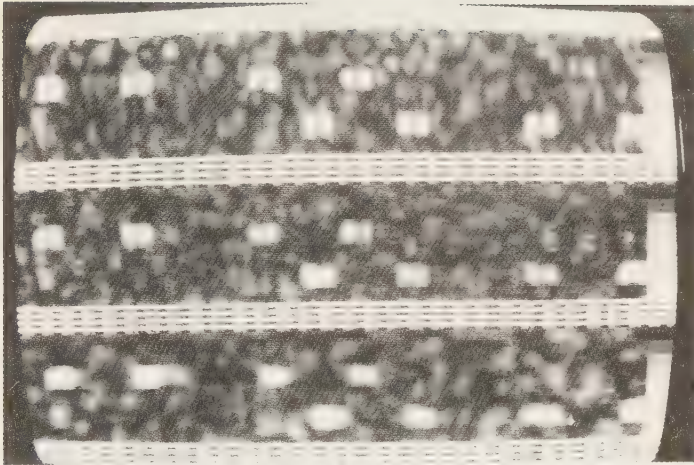


Fig. 3.17 a) Zoomed ideal B-scan image.
 b) Frequency dependent compensated B-scan image with
 $\text{SNR}_{\text{surface}} = 40\text{dB}$ (the filter in Fig. 3.6a is used).
 c) TGC B-scan image $\text{SNR}_{\text{surface}} = 40\text{dB}$.

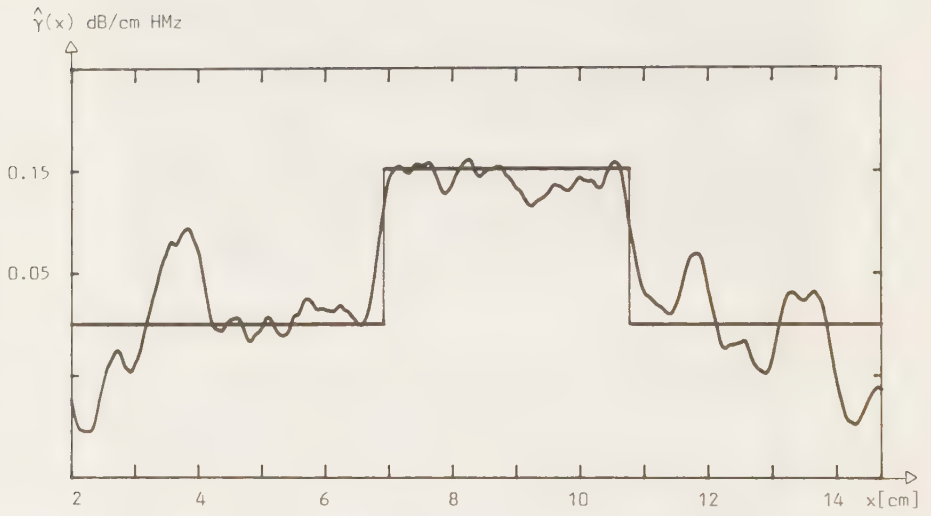


Fig. 3.18 The averaged estimated attenuation coefficient $\hat{\gamma}(x)$ together with the true attenuation coefficient $\gamma(x)$.

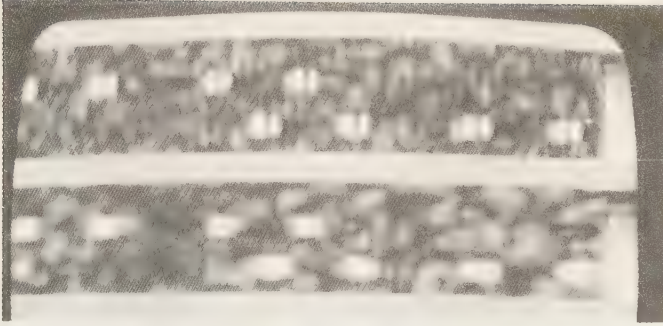


Fig. 3.19. a) Frequency compensated B-scan image with varying attenuation according to Fig. 3.18. $\text{SNR}_{\text{surface}} = 60\text{dB}$.
 b) TGC B-scan image with varying attenuation according to Fig. 3.18. $\text{SNR}_{\text{surface}} = 60\text{dB}$.

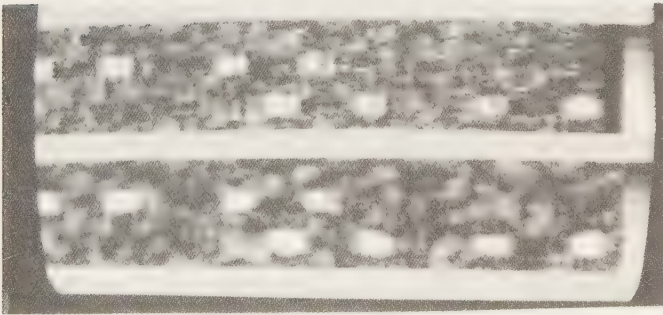


Fig. 3.20. a) Frequency compensated B-scan image with varying attenuation according to Fig. 3.18. $\text{SNR}_{\text{surface}} = 40\text{dB}$.
 b) TGC B-scan image with varying attenuation according to Fig. 3.18. $\text{SNR}_{\text{surface}} = 40\text{dB}$.

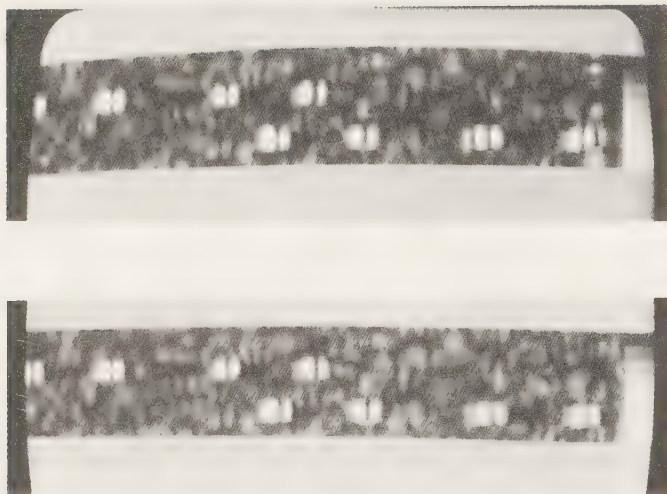


Fig. 3.21. Comparison of frequency compensated images using
a) filter sequence in Fig. 3.6d
b) filter sequence in Fig. 3.7d
on simulated signals with $\text{SNR}_{\text{surface}} = 80\text{dB}$.



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